THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXIV JULY 1906 NUMBER 1

ON THE LIGHT-VARIATIONS OF ASTEROIDS AND SATELLITES

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§ 1. It has often been suggested that the light-changes of those variable stars which are regularly periodic may be due to the existence of spots on their surfaces, which are hidden or brought into view as the star rotates about its axis. The same explanation has been given more plausibly for the variability shown by certain satellites and asteroids, for in this case the only rival hypothesis is that which ascribes the light-changes to the departure of the body from a spherical form.

It may therefore be worth while to discuss some results of these hypotheses, and consider (1) what is the character of the light-curve produced by the rotation of an arbitrarily spotted body, and (2) how far it is possible to reason backward from such a light-curve to the spots which produce it.

§ 2. The simplest case of a spotted body is a self-luminous sphere without absorbing atmosphere. In this case the apparent brightness of any part of the surface is independent of its inclination to the line of sight, so that a sphere of constant intrinsic brightness appears as a uniformly illuminated disk.

The case of a body shining by reflected light is greatly complicated by the effects of phase; but if we confine ourselves to planets close to opposition, the appearance of the full Moon shows that

the hypothesis of a uniformly luminous disk is not very far from representing the facts, at least for a body devoid of atmosphere. Bodies with a dense atmosphere, like the Sun and *Jupiter*, are likely to appear darker near the limb, and our theory must be modified to include them.

If B denotes the intrinsic brightness of an element $d\sigma$ of the surface of the sphere, γ the angle between the (outward) normal to the sphere at this point and the line of sight, and R the distance of the element from the observer, the apparent area of the element will be $\dfrac{d\sigma \cos\gamma}{R^2}$, and the light which the observer receives from it will be

$$dL = \frac{B \cos\gamma\, d\sigma}{R^2}, \tag{1}$$

provided that there is no absorbing atmosphere.

If there is such an atmosphere, its effective thickness will increase as we approach the limb, and the percentage of transmitted light will decrease. This percentage will be a function of γ alone. If we denote it by A, we shall have for the light received from the element $d\sigma$

$$dL = \frac{AB \cos\gamma\, d\sigma}{R^2}. \tag{1a}$$

This expression may also be made to include the effects of any brightening or darkening of the limb due to peculiarities of the law of diffuse reflection (which is known to vary from planet to planet).

To find the total light L which the observer receives from the sphere, we must integrate the expressions (1) or (1a) over the visible portion S of the sphere (that is, the region in which $\cos\gamma$ is positive), and thus we obtain the equations

$$L = \int_S \frac{B \cos\gamma\, d\sigma}{R^2}, \tag{2}$$

which holds good when there is no absorption; and

$$L = \int_S \frac{AB \cos\gamma\, d\sigma}{R^2}, \tag{2a}$$

which includes the effect of absorption.

These expressions are perfectly general, and hold good for any point outside the sphere.

We will now confine ourselves to the consideration of very distant points of observation for which the radius of the sphere is negligible in comparison with its distance. We may then without sensible error disregard the variations in the distance and direction of different parts of the sphere from the observer, so that in (2) we may treat R as constant and equal to the distance of the sphere's center, while γ becomes the angle at the center between lines drawn to the observer and to any given element. Our equations then become

$$L = \frac{1}{R^2} \int_S B \cos \gamma \, d\sigma \, , \tag{3}$$

or, if we include the effects of an atmosphere,

$$L = \frac{1}{R^2} \int_S A B \cos \gamma \, d\sigma \, , \tag{3a}$$

where the integrals extend over the visible hemisphere, for which $\cos \gamma$ is positive.

§ 3. Let us now choose any system of polar co-ordinates ρ, θ, ϕ, fixed with reference to the sphere, and let r denote the radius of the sphere. The brightness B at any point of the sphere's surface will be a function of its co-ordinates θ, ϕ. We may show this explicitly by writing it in the form $B\,(\theta, \phi)$.

If we denote the co-ordinates of the observer (referred to this system) by R, θ_0, ϕ_0, the light which he receives from the sphere will depend only on these three quantities. From (3) we see that it may be expressed in the form $L = \frac{1}{R^2} j\,(\theta_0, \phi_0)$.

Since we have

$$d\sigma = r^2 \sin \theta \, d\theta \, d\phi \, ,$$

and since from the definition of γ it follows that

$$\cos \gamma = \cos \theta \cos \theta_0 + \sin \theta \sin \theta_0 \cos (\phi - \phi_0) \, ,$$

it is clear that the equations (3) or (3a) enable us to determine the value of $j\,(\theta_0, \phi_0)$ for any particular values of θ_0 and ϕ_0, when the function $B\,(\theta, \phi)$ is known. We wish, however, to obtain general relations between the functions $j\,(\theta_0, \phi_0)$ and $B\,(\theta, \phi)$. Since both these functions are given by their values over the surface of a sphere, it is natural to seek to express them in terms of surface spherical harmonics.

From physical considerations $B(\theta, \phi)$ must always be finite and positive (or zero). It will be expansible in a spherical harmonic series if it satisfies Dirichlet's conditions; that is, if it has only a finite number of maxima, minima, and points or lines of discontinuity. These conditions impose no limitation of practical importance upon our function, and we shall hereafter suppose that they are satisfied, so that there exists an expansion of the form

$$B(\theta, \phi) = Y_0 + Y_1(\theta, \phi) + Y_2(\theta, \phi) \ldots + Y_n(\theta, \phi), + \ldots \quad (4)$$

where Y_n is a surface spherical harmonic of the nth order.

§ 4. It will be convenient at this point to recall certain properties of such expansions. The spherical harmonic Y_n is the sum of $2n+1$ terms involving Legendre's functions of the form

$$Y_n(\theta, \phi) = A_{n,0} P_n(\cos\theta) + \sum_{m=1}^{m=n} (A_{n,m} \cos m\phi + B_{n,m} \sin m\phi) P_n^m(\cos\theta). \quad (5)$$

The numerical coefficients are given by the definite integrals

$$\left.\begin{aligned} A_{n,0} &= \frac{2n+1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\pi} B(\theta, \phi) P_n(\cos\theta) \sin\theta \, d\theta \\ \begin{matrix} A_{n,m} \\ B_{n,m} \end{matrix} &= \frac{2n+1}{2\pi} \int_0^{2\pi} d\phi \int_0^{\pi} B(\theta, \phi) P_n^m(\cos\theta) \begin{matrix} \cos \\ \vdots \\ \sin \end{matrix} m\phi \sin\theta \, d\theta \end{aligned}\right\} \quad (6)$$

Y_0 is a constant, independent of θ and ϕ.

The series (4) is thus uniquely determined. When B (θ,ϕ) satisfies Dirichlet's conditions, it is convergent. Its sum, however, will in general present discontinuities, like that of a Fourier series. But if we multiply each term of such a series by a quantity which is numerically less than $\frac{A}{n}$, where A is a constant, the series so obtained will have a sum which is continuous for all values of θ and ϕ.

If the multiplier is always numerically less than $\frac{A}{n^2}$, the sum of the new series will be continuous, and will possess continuous derivatives of the first order for all values of θ and ϕ.

If we take any two antipodal points, at the extremities of a diameter of the sphere, the values of $Y_0, \ldots Y_2$ and all the even harmonics will be equal at the two points, while those of the odd harmonics Y_1, $Y_3 \ldots$ will be numerically equal, but of opposite sign.

Finally, we may express $Y_n(\theta, \phi)$ as a function of the angle γ, already defined, by introducing another angle λ, which may be defined geometrically as the position angle of any point on the apparent disk of the sphere, relative to its center, as seen by the observer, r, γ, and λ will then form a system of polar co-ordinates, and there will exist an expansion of the form,

$$Y_n(\theta, \phi) = a_0 P_n(\cos\gamma) + \sum_1^n (a_m \cos m\lambda + b_m \sin m\lambda) P_n^m(\cos\gamma) \; . \quad (7)$$

If in this expression we set $\gamma = 0$, then θ and ϕ will have the values θ_0 and ϕ_0, while $P_n(\cos\gamma) = 1$, and $P_n^m(\cos\gamma) = 0$ for all values of m, and so we must have

$$a_0 = Y_n(\theta_0, \phi_0) \; . \quad (8)$$

§ 5. We are now in a position to evaluate the function $L(\theta_0, \phi_0)$. Substituting in (3) $d\sigma = r^2 \sin\gamma \, d\gamma \, d\lambda$, and considering only the term $Y_n(\theta, \phi)$ in B, we find for the corresponding term in L,

$$\frac{r^2}{R^2}\int_0^{2\pi} d\lambda \int_0^{\frac{\pi}{2}} \left\{ Y_n(\theta_0, \phi_0) P_n(\cos\gamma) + \sum_1^n (a_m \cos m\lambda + b_m \sin m\lambda) P_n^m \cos\gamma \right\} \cos\gamma \sin\gamma \, d\gamma \, ,$$

where the limits correspond to the condition that the integral shall be taken over the visible hemisphere.

Integrating with respect to λ, all the terms except the first vanish and we obtain

$$\frac{2\pi r^2}{R^2} Y_n(\theta_0, \phi_0) \int_0^{\frac{\pi}{2}} P_n(\cos\gamma)\cos\gamma \sin\gamma \, d\gamma \; . \quad (9)$$

Setting $\cos\gamma = x$, the integral becomes

$$\int_0^1 P_n(x) x \, dx \; .$$

This is a known integral. Its value is $\frac{1}{2}$ when $n=0$; $\frac{1}{3}$ when $n=1$; $\frac{1}{8}$ when $n=2$; zero when n is odd and greater than 1; and $\frac{1, 3, 5 \ldots (n-3)}{2, 4, 6 \ldots (n+2)}(-1)^{\frac{n}{2}+1}$ when n is even and greater than 2. Hence, collecting the terms of L, we obtain

$$L(\theta_0, \phi_0) = \frac{\pi r^2}{R^2}\left(Y_0 + \frac{2}{3}Y_1(\theta_0, \phi_0) + \frac{1}{4}Y_2(\theta_0, \phi_0) - \frac{1}{24}Y_4(\theta_0, \phi_0) + \frac{1}{64}Y_6(\theta_0, \phi_0) - \frac{1}{128}Y_8(\theta_0, \phi_0) \ldots\right) \quad (10)$$

Since the spherical harmonics Y are completely determined when the surface-brightness $B(\theta, \phi)$ is known, the equation (9) contains the complete solution of our problem. We may deduce from it several important properties of this solution:

(1) We may write the coefficient of Y_n in the form

$$\pm\frac{\pi r^2}{R^2} \cdot \frac{3, 5, 7 \ldots (n-3)}{4, 6, 8 \ldots (n-2)} \cdot \frac{1}{n(n+2)},$$

which shows that it is always numerically less than $\frac{\pi r^2}{R^2 n^2}$. It follows that the series (9) is absolutely and uniformly convergent, and that the function L which it represents is continuous, as well as its first derivatives, for all values of θ_0 and ϕ_0. Hence the light-curve due to the rotation of the sphere about any axis must be continuous, and free from abrupt changes of direction, even though the distribution of brightness on its surface is discontinuous.

(2) Since the odd harmonics Y_3, Y_5, etc., do not appear in L, it is clear that a great variety of distributions of brightness on the sphere may give rise to the same light-curve.

(3) Let us now take the axis of rotation as the axis of our polar co-ordinates θ, ϕ. Then, as the sphere rotates (while the positions of its center and the observer remain the same), θ_0 will be constant, and equal to the inclination of the axis of rotation to the line of sight, while ϕ_0 will increase uniformly with the time. If ω is the angular velocity of rotation, we will have $\phi_0 = \omega(t - t_0)$, t_0 being the time when the meridian $\phi = 0$ crosses the center of the disk.

Introducing these into (9), we obtain the expression for the light received by the observer as a function of the time—that is, the equation of the light-curve.[1] Each of the spherical harmonics Y_n becomes a finite Fourier series in ϕ_0, whose coefficients are functions of θ_0, terminating with the terms in $\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} n\phi_0$. Since the series (9)

[1] By "light-curve" throughout the present discussion is meant the curve which gives the light itself as a function of the time and not that which gives the corresponding stellar magnitude.

is absolutely convergent, we may rearrange it, and collect the terms involving like multiples of ϕ_o. We thus obtain the equation of the light-curve, in the form of the Fourier series

$$L=\frac{2\pi r^2}{R^2}\begin{pmatrix} C_o+C_1\cos\omega(t-t_o)+C_2\cos 2\omega(t-t_o)\ldots\ldots \\ +D_1\sin\omega(t-t_o)+D_2\sin 2\omega(t-t_o)\ldots\ldots \end{pmatrix}, \tag{11}$$

where the coefficients C and D are functions of θ_o of the form

$$\begin{aligned} C_o(\theta_o)&=\frac{A_{1,o}}{3}\cos\theta_o+\sum_{n=o}^{n=\infty}a_{n,o}P_n(\cos\theta_o) \\ C_1(\theta_o)&=\frac{A_{1,1}}{3}\sin\theta_o+\sum_{n=2}^{n=\infty}a_{n,1}P_n^1(\cos\theta_o) \\ C_2(\theta_o)&=\sum_{n=2}^{n=\infty}a_{n,2}P_n^2(\cos\theta_o)\ , \end{aligned} \tag{12}$$

and in general

$$C_m(\theta_o)=\sum_{n=m}^{n=\infty}a_{n,m}P_n^m(\cos\theta_o)\ ,$$

where the summations extend only over *even* values of n. The expressions for D_1, D_2, etc., are obtained by introducing the numerical constants $b_{n,m}$ in place of $a_{n,m}$.

Now, $P_n^m(\cos\theta_o)$ is an even or odd function of $\cos\theta_o$, according as $n-m$ is even or odd. It follows that $C_m(\theta_o)$ is an odd or even function of θ_o according as m is odd or even. Hence we have, except for C_o and C_1,

$$\begin{aligned} C_m(\theta_o)&=(-1)^m C_m(\pi-\theta_o) \\ D_m(\theta_o)&=(-1)^m D_m(\pi-\theta_o)\ . \end{aligned} \tag{13}$$

The corresponding relations for C_o and C_1 are

$$\left.\begin{aligned} C_o(\theta_o)-C_o(\pi-\theta_o)&=\frac{2}{3}A_{1,o}\cos\theta_o \\ C_1(\theta_o)+C_1(\pi-\theta_o)&=\frac{2}{3}A_{1,1}\sin\theta_o\ . \\ D_1(\theta_o)+D_1(\pi-\theta_o)&=\frac{2}{3}B_{1,1}\sin\theta_o \end{aligned}\right\} \tag{14}$$

When $\theta_o=\frac{\pi}{2}$ (that is, when the observer is in the equatorial plane), all the odd coefficients except C_1 and D_1 must vanish, and all the even ones except C_o must be at a maximum or a minimum.

When $\theta_\circ = o$, $P_n(\cos\theta_\circ) = 1$ and $P_n^m(\cos\theta_\circ) = o$, so that all the coefficients vanish except $C_\circ$, and the light is constant. This is geometrically obvious, as in this case the observer always sees the same hemisphere.

§ 6. The case of a spotted sphere with an absorbing atmosphere may be discussed in a similar fashion, starting from equation (3*a*) instead of (3), the only difference being the presence of the transmission-factor A.

When we come to integrate the equation (9), we may expand $A\cos\gamma$ (which is a function of γ alone) in a series of Legendre functions.

$$A\cos\gamma = \alpha_\circ + \alpha_1 P_1(\cos\gamma) \ldots\ldots + \alpha_n P_n(\cos\gamma) \ldots\ldots .$$

We shall then be led to integrals of the form $\int_\circ^1 P_n(x) P_m(x) dx$. The value of such an integral is finite and different from zero if $n-m$ is odd, but is zero if $n-m$ is even (except for $n=m$, when it is $\frac{1}{2n+1}$).

We thus obtain the coefficient of $Y_n(\theta_\circ, \phi_\circ)$ in the expansion of L, in terms of the constants $\alpha_\circ \ldots\ldots \alpha_n \ldots\ldots$ so that, when the law of absorption is known, the expansion of L is completely determined. In this case, however, the expansion will in general contain the odd harmonics Y_3, Y_5, etc., and consequently the relations (13) and (14) between the coefficients of different light-curves will no longer hold good.

If therefore we know the light-curves of a rotating sphere for a sufficient range of values of $\theta_\circ$, we can determine whether or not it has an absorbing atmosphere (or some other peculiarity of surface which has a similar effect on the appearance of its disk).

It can be proved that, if the transmission-factor A is a continuous function of γ, the coefficient of Y_n in the expansion of L will be of the order of $\frac{1}{n^2}$, so that it will still be true that the light-curve can have no sharp angles.

§ 7. We may extend our results to include the case of any convex solid, arbitrarily spotted.

By a convex solid we mean one such that any tangent plane

meets its surface in only one real point. (This definition includes solids bounded by portions of several intersecting convex surfaces, but excludes polyhedra with plane or conical faces.) On such a surface there will be one, and only one, point where the (outward) normal has a given direction.

Just as in § 2, we shall have, for the light received at a distant point,

$$L=\frac{1}{R^2}\int_S B\cos\gamma\, d\sigma\ , \tag{3}$$

where $d\sigma$ is an element of the surface, and γ is the angle between the outward normal to this element and the line of sight, and the integral extends over all elements for which $\cos\gamma$ is positive.

Now, let θ, ϕ be the polar co-ordinates corresponding to the direction of this normal. Since the surface is convex, their values define uniquely the position of the given element, and hence the value of B, which may therefore be regarded as a function of θ, ϕ. The Gaussian curvature C of the surface at this point will also be a function of θ, ϕ. But by the definition of this kind of curvature we have

$$Cd\sigma=\sin\theta\, d\theta\, d\phi\ . \tag{15}$$

Hence we may write (3) in the form

$$L=\frac{1}{R^2}\int_S \frac{B}{C}\cos\gamma\sin\theta\, d\theta\, d\phi\ .$$

This differs from the integral discussed in § 3 only by the presence of the denominator C, and we may proceed just as we did then, except that $\frac{B}{C}$ is the function which is expanded in the spherical harmonic series (4).

The resulting values of L will be identical with those given by a sphere of radius r and surface brightness $\frac{r^2B}{C}$ (corresponding points on the sphere and surface being those for which θ, ϕ are equal).

It thus appears that there is no gain in generality in passing from a spotted sphere to a spotted convex surface—that is, in assuming that the curvature is variable, as well as the surface brightness.

On the other hand, if we assume that the surface brightness is constant, and only the curvature variable, we lose somewhat in generality.

The boundary of our solid is necessarily a closed surface. Hence its orthogonal projection on any plane must be zero. This condition gives us $\int_S d\sigma \cos\gamma = 0$, where γ is the angle between the normals to the element $d\sigma$ and the plane, and the integral extends over the whole surface. Introducing the variables θ and ϕ, and expanding $\frac{1}{C}$ in a spherical harmonic series, we easily find that this equation will be satisfied when, and only when, the expansion of $\frac{1}{C}$ contains no harmonic of the first order. Since B is constant, it follows that the expansion of L contains no harmonic of the first order. By § 5 all the other odd harmonics are absent, so that the expansion contains only even harmonics and the relations (13) hold good for all values of n, including 0 and 1.

This may also be proved geometrically. Since the surface is of uniform brightness, the light received at any distant point will be proportional to the area of the celestial sphere which the body appears to cover. For two points at equal (great) distances in opposite directions, the apparent areas of the body will be the same, and the values of L identical. It follows at once that the expansion of L contains no odd harmonics.

§ 8. This proof has the advantage that it is available even if the body is not convex. It may even consist of several separate parts (all of the same brightness), provided the system rotates like a rigid body.

In this case, unlike the previous ones, the light-curve may have abrupt changes of direction. For example, a cube, rotating about an axis parallel to one of its edges and seen from a point in the equatorial plane, gives a light-curve consisting of segments of different sine-curves, which cut at sharp angles. There may be such angles in the light-curve even when the surface of the body itself is smooth—for example, when it consists of two equal spheres, which eclipse one another centrally.

A spotted body which is not convex may give us still greater variety. It is clear, however, that the light-curve itself must always be continuous, for otherwise a rotation through an infinitesimal

angle must produce a finite change in the light received, which is impossible if the area and brightness of the surface are finite.

Hence in all cases L may be expanded in a spherical harmonic series in θ, ϕ, but in this series the terms of both even and odd orders will in general be present. It will still be possible to obtain the equation of the light-curves in the form (11), where the coefficients are functions of θ_0, but no general relations will exist between the values of the coefficients for different values of θ_0, except that, in all cases, the light will be constant if the body is viewed from a point on its axis of rotation, so that all the coefficients except C_0 must vanish when $\theta_0 = 0$ or $\theta_0 = \pi$.

§ 9. We may now consider the inverse problem: Given the observed light-curves of a body, to determine the position of its axis of rotation and the character of its surface.

In the case of a star this problem is indeterminate. All that we can know about the inclination θ_0 of the axis of rotation to the line of sight is that it is not zero, if the star is variable (provided, of course, that the variability is to be explained by the star's rotation). Even if we knew the value of θ_0, we could not hope to find out much about the form or spottedness of the star's surface, for our knowledge of the values of the function $L\ (\theta_0, \phi_0)$, for the given value of θ_0, does not inform us how it behaves for other values of this variable.

It is only when we may start with a much less generalized hypothesis (for example, the eclipse theory for *Algol* variables) that we can hope to gain much information about the surface conditions of variable stars.

§ 10. The case of an asteroid is much more promising. We may observe it from all directions, approximately in the plane of its orbit, and the values of θ_0 will have a range equal to twice the inclination of its equator to this plane. In what follows we shall suppose that we know the light-curves of the planet (corrected for the influence of its varying distance from the Earth and Sun) for a series of oppositions, well distributed around the orbit.

We have first to find the position of its equator. We shall take the planet's equator and orbit as fundamental planes, and, for brevity, shall use the terms "longitude," "right ascension," and

the like, as they would be used by an observer living on its surface. If we suppose him to be situated on the initial meridian (from which longitudes on the surface are measured), then θ_0 and ϕ_0 will be the north polar distance and hour-angle of the Earth, as seen by this observer.

Let Ω denote the longitude of his vernal equinox, measured from some fixed point in the orbit plane (e. g., the perihelion), and i the inclination of the equator to this plane. Let l be the Earth's longitude, measured from the same point, and b its latitude, and let ψ be the Earth's right ascension (referred to this equator and equinox). l and b are given by our observations; b will usually be small, and for the present we shall neglect it. We shall then have

$$\begin{aligned} \cos\psi \sin\theta_0 &= \cos(l-\Omega) , \\ \sin\psi \sin\theta_0 &= \sin(l-\Omega) \cos i ; \\ \cos\theta_0 &= \sin(l-\Omega) \sin i . \end{aligned} \tag{17}$$

If, then, we can determine the values of ψ and θ_0, we can find Ω and i.

Now, the form of the light-curve depends only upon θ_0, while its phase varies with ϕ_0. From the last of the equations (17) we see that θ_0 reaches any given value for two different values of l. Calling them l_1 and l_2, we have, for all values of θ_0

$$l_1 + l_2 = 2\Omega + \pi . \tag{18}$$

That is, the points whose longitudes are l_1 and l_2 are equidistant from the point whose longitude is $\Omega + \frac{\pi}{2}$ (the solstitial point). The light-curves of the planet for oppositions in these two longitudes will be identical in form.

If, then, we arrange the observed light-curves in order according to the opposition-longitudes l, it will be an easy matter to determine the solstitial point, and hence the value of Ω.

For any two oppositions for which the values of l differ by 180°, the values of $\cos\theta_0$ will be equal and of opposite sign. If the planet's surface is of uniform brightness (whatever its form), we see from (13) that the light-curves at these two oppositions must be identical. If it is convex, and has no absorbing atmosphere, it follows from (14) that the two light-curves can differ only in the terms involving C_0, C_1, and D_1.

We therefore have the following three rules for determining the character of the planet's surface:

I. If it is not possible to find a point such that the light-curves at oppositions equidistant from it in longitude are identical, then the planet's variability cannot be accounted for by the rotation of any body which permanently maintains its form, markings, and axis of rotation.

II. If the light-curves at oppositions in diametrically opposite longitudes are not identical, the planet must be spotted, or have an absorbing atmosphere.

III. If the difference between the light-curves at two such oppositions cannot be reduced to a simple sine-curve, by a proper choice of initial epochs for the two curves, then the planet cannot be a convex body, unless it has an absorbing atmosphere, or some equivalent surface peculiarity.

We have now to determine the inclination i. Consider two oppositions, in longitudes l_1 and l_2, for which the light-curves are the same. Similar phases of the two curves will correspond to the times of transit of the Earth across the meridian of our imaginary observer on the planet. If ψ_1 and ψ_2 are the (planeto-centric) right ascensions of the Earth at the two oppositions, the angle through which the planet has turned between the times of corresponding phases of the light-curve at the two oppositions will be some integral number of revolutions *plus* $\psi_2 - \psi_1$. If we know the rotation period accurately, we can therefore determine $\psi_2 - \psi_1$. But from (17) we have $\tan \psi_1 = \cos i \tan (l_1 - \Omega)$, $\tan \psi_2 = \cos i \tan (l_2 - \Omega) = -\tan \psi_1$, whence

$$\cos i = \frac{\tan \frac{1}{2}(l_2 - l_1)}{\tan \frac{1}{2}(\psi_2 - \psi_1)} \,. \tag{19}$$

Every such pair of oppositions will give us a determination of $\cos i$. It is easy to show that the most favorable pairs are those for which $l_2 - l_1$ is near 90° or 270°.

To determine the accurate value of the rotation period we may use either two oppositions in the same longitude for which the values of θ_0 and ψ are equal, or two near the two nodes of the equator on the orbit, for which $\theta_0 = 0$ and $\psi_2 - \psi_1 = 180°$, whatever the value of i.

Since i is given by its cosine, it is clear that small inclinations cannot be accurately determined from the observations. Even if $i=30^\circ$, the maximum difference between $\psi_2-\psi_1$ and l_2-l_1 is only about 9°, corresponding to a displacement of similar phases of the light-curve by $\frac{1}{40}$ of the period. In any case, the sign of i remains indeterminate, as it is clear geometrically that it should do.

When the Earth's latitude b is not negligible, the equations (17) become much more complex. But we may reduce this case to the one already studied, as soon as we have found two oppositions with identical light-curves, by choosing as our fundamental plane one which passes through the places of the Earth at these two oppositions, and measuring longitude anew from some point in this plane. The equations (18) and (19) then give the position of the planet's equator, relative to this plane.

§ 11. We have now to find out what we can about the character of the planet's surface. We will first consider the case when Rule III of the preceding section shows that the light-changes can be accounted for by the rotation of a spotted sphere. Referring to § 5, we see that, in order to find an expression for the surface brightness B, we must expand $L(\theta_0, \phi_0)$ in a spherical harmonic series.

Knowing the position of the planet's equator, and its exact rotation-period, we may find for each opposition an epoch t_0, when the same initial meridian crosses the center of the apparent disk, and so obtain the equation of the light-curve in the form (11), where we may set $\omega(t-t_0)=\phi_0$. The value of θ_0 is known for each curve, so that we may express the coefficients C and D as functions of this variable. The conditions (13) will be satisfied, since they are equivalent to Rule III. The equations (14), with the aid of (17), will then give us a new determination of the node and inclination of the planet's equator, which must agree with the previous one. They serve also to determine completely the harmonic Y_1, in the expansion (10) of L.

Now, θ_0 may have any value included between $\frac{\pi}{2}+i$ and $\frac{\pi}{2}-i$, but it cannot exceed these limits. We therefore know the function $L(\theta_0, \phi_0)$, not over the whole sphere, but over an equatorial zone, whose limiting latitudes are $\pm i$. But we must have its values over

the whole sphere in order to determine the spherical harmonic series (10) uniquely. All that we know about its behavior in the rest of the sphere is (1) that it itself, and its first derivatives, must be everywhere finite and continuous; and (2) that the difference between its values at diametrically opposite points must be equal to the difference of the values of Y_1 at these points (since the higher odd harmonics are absent). We may therefore assign to it any arbitrary system of values, satisfying the first condition, over one of the two polar regions, when the second condition will determine it over the opposite polar region.

To every such assigned system of values there will correspond a different harmonic expansion of the form (10), and all of these expansions will perfectly represent the observed light-curves, while each of them will lead us to a different distribution of brightness on the planet's surface. The form of the expansion will be completely determined by the observations only when $i=\frac{\pi}{2}$; that is, when the planet's axis of rotation lies in the plane of its orbit.

Knowing the expansion of L, we may at once obtain that of B, by multiplying each term of the former by a suitable constant factor. Comparing (10) and (4), we see, however, that the expansion of B is not completely determined, for the odd harmonics Y_3, Y_5 . . . . which appear in it do not affect L at all, and so we have no means of finding their values.

If, then, we write the result obtained from our observations in the form

$$L(\theta_0, \phi_0)=S_0+S_1(\theta_0, \phi_0)+S_2(\theta_0, \phi_0)+S_4(\theta_0, \phi_0)+S_6(\theta_0, \phi_0)+ \dots \quad (20)$$

We shall have

$$B(\theta,\phi)=\frac{R^2}{\pi r^2}\left\{\begin{aligned} &S_0+\frac{3}{2}S_1(\theta,\phi)+4S_2(\theta,\phi)-24S_4(\theta,\phi)+64S_6(\theta,\phi)+ \dots \\ &\qquad +Y_3(\theta,\phi)+Y_5(\theta,\phi)+Y_7(\theta,\phi)+ \dots \end{aligned}\right\} \quad (21)$$

where Y_3, Y_5, etc., denote spherical harmonics of the orders 3, 5 . . . . , which may be chosen at random.

This is the complete solution of our problem. It shows that, even in the most favorable case, there is an infinite variety of distributions of brightness on an asteroid's surface, which will account for its observed variations in light.

§ 12. The solution as it stands is, however, only formal, for we have taken no account of the physically necessary condition that the surface-brightness B can never be negative. It may be that, although the series (20) gives always positive (and so physically possible) values for L, the series (21) gives negative values for B.

We may distinguish two cases, if we set $B=B_1+B_2$, where B_1 denotes the sum of the odd, and B_2 that of the even harmonics. B_2 is completely determined by the observations, while in B_1 only the term $\frac{3}{2}S_1$ is known. Now, B_2 may be defined as half the sum of the values of B at any point and the diametrically opposite point, and B_1 as half their difference. If then B_2 is negative at any point, B must be negative either at this point or the opposite point, whatever the value of B_1 may be.

In this case it is physically impossible to account for the observed variability by the rotation of a spotted sphere.

If B_2 is everywhere positive, or zero, B will be so too, provided the numerical value of B_1 is never greater than that of B_2.

If $\frac{3}{2}S_1$ is never greater than B_2, this condition may be satisfied by setting $Y_3=Y_5 \ldots . = 0$. When this is not the case, it may still be possible to assign such values to Y_3, Y_5, etc., that B_1 is not greater than B_2.

Let us denote the expression for B thus found by B_0. It will be a particular solution of our problem. The general solution may be expressed in the form $B=B_0+B_3$, where B_3 denotes any sum of odd harmonics, of order greater than 1, such that $B_0+B_3 \geqq 0$ for all points on the sphere.

The problem is in general still indeterminate. In certain cases it may not be so. Suppose, for example, that $L=0$ for some position of the observer. Then the whole hemisphere which is visible from his direction must be dark. That is, any physically possible solution must have $B=0$ over this hemisphere. The difference of two such solutions vanishes over the hemisphere. But since it consists wholly of odd harmonics, it must vanish over the whole sphere; that is, the two solutions are identical.

If the rules of § 10 show that the planet has an absorbing atmos-

phere, we may assume a law of absorption, and then, by reversing the reasoning of § 6, determine the markings on its surface. In this case all the harmonics, odd and even, will usually be determinate. But as we do not know what the actual law of absorption is, we are no better off than before.

§ 13. Instead of assuming that our planet is spherical, we may assign to it any other convex form. The expansion (21) will then give the values of $\frac{B}{C}$ at every point of the surface, which serve to define the markings.

Since the surface is convex, C is always positive. For a surface without sharp edges, C will everywhere be finite and the conditions for a physically possible solution will be those discussed in the last section. For a body with sharp edges there will be certain ranges of value of θ, ϕ for which C is infinite, and a physically possible solution must give $\frac{B}{C}=0$ for all such values of the variables, in addition to satisfying the previous conditions.

When the first harmonic is absent in the expansion of $\frac{B}{C}$, we might assume that B is constant, and seek to account for the light-changes by the form of the surface alone. But this leads to difficult problems in the theory of surfaces,[1] and will not be attempted here.

§ 14. We have so far confined ourselves to the study of the light-curves of the planet at opposition. At other phases the light received from a given element $d\sigma$ will depend not only on γ, but also on the angle of incidence i of the Sun's rays, and the expression for the light received by the observer from this element will be $dL=\frac{B}{R^2}f(i, \gamma)d\sigma$, where $f(i, \gamma)$ vanishes when either $\cos i$ or $\cos \gamma$ is zero. To obtain the total light L, this must be integrated over that part of the surface for which $\cos i$ and $\cos \gamma$ are both positive.

It will still be true that any two convex surfaces for which the values of $\frac{B}{C}$ at corresponding points are the same will give identical values of L. But in this case the odd harmonics Y_3, Y_5, etc., in

[1] For example, the equation C=constant leads not only to the sphere, but to all the surfaces applicable to a sphere.

the expansion of $\frac{B}{C}$ will in general influence the value of L, and from this their values may sometimes be determined.

§ 15. We may summarize the results of this discussion as follows:

If a variable asteroid has been observed at a series of oppositions in all parts of its orbit:

(1) We can determine by inspection of its light-curves whether or not they can be accounted for by its rotation alone, and, if so, whether the asteroid (*a*) has an absorbing atmosphere, (*b*) is not of a convex form, (*c*) has a spotted surface, or whether these hypotheses are unnecessary.

(2) It is always possible (theoretically) to determine the position of the asteroid's equator, (except that the sign of the inclination remains unknown).

(3) It is quite impossible to determine the shape of the asteroid. If any continuous convex form is possible, all such forms are possible.

(4) In this case we may assume any such form, and then determine a distribution of brightness on its surface which will account for the observed light-curves. This can usually be done in an infinite variety of ways.

(5) The consideration of the light-curve of a planet at phases remote from opposition may aid in determining the markings on its surface, but cannot help us to find its shape.

Princeton University Observatory,
April 28, 1906.

POLARIZATION AND SELECTIVE REFLECTION IN THE INFRA-RED SPECTRUM

By A. H. PFUND

INTRODUCTION

Object and general survey of the work.—The intimate relationship existing between the refractive index, extinction coefficient, and reflecting power of an absorbing medium was first pointed out by Cauchy in his well-known formulæ for metallic reflection. That these formulæ do actually give a true account of existing conditions has been proved in recent years by Minor,[1] Pflueger,[2] Edmunds,[3] and others, for conductors as well as for insulators. In deducing his formulæ, Cauchy made the very natural assumption that the intensity of the light, as it penetrated into the medium, fell off according to some exponential law. Now, although this assumption leads to formulæ which are verified by experiment, no insight is obtained into the actual mechanism of reflection, and, in fact, up to the present time a satisfactory theory bearing upon this subject is wanting. It was with the object of collecting data which, it was hoped, might be of some help in solving the problem, that the present work was taken up. For the sake of greater clearness, I wish to give in the following a brief survey of the several investigations carried out.

As indicated by the title, the work naturally falls under two headings:

1. Polarization in the infra-red.
2. Selective reflection in the infra-red.

Heretofore investigators working with polarized radiations in the infra-red have simply assumed that these radiations were polarized, without actually proving them to be so. Therefore the first point to be taken up was to show definitely that these radiations were susceptible of polarization. In the course of the work a new polarizer and analyzer were discovered which are believed to be a decided

[1] *Ann. d. Phys.*, **10**, 581, 1903.
[2] *Ibid.*, **65**, 214, 1898.
[3] *Phys. Rev.*, **18**, 193, 385, 1904.

improvement upon the forms now in use. With these new instruments it was readily proved, as far as the experiment could be carried (i. e., up to 13 μ), that infra-red radiations can be polarized by reflection.

One of the distinguishing properties of a metal is its ability to convert plane into elliptically polarized light by reflection. It was thought of interest to investigate whether an insulator reflecting metallically in the infra-red would also have this property. The substance chosen was Iceland spar, and it was shown conclusively that the conversion of plane into elliptically polarized light does take place—as might have been expected.

Originally it was intended to determine the refraction and extinction curves for Iceland spar within the region of metallic reflection by a katoptric method which involved a determination of the constants of elliptically polarized light. The method employed was the well-known Brewster's double mirror method as used by Quincke,[1] Pflueger (*loc. cit.*), and others. It was shown by the use of silver mirrors that such measurements can easily be carried out into the infra-red. Unfortunately, however, in the case of Iceland spar the amount of energy reaching the radiometer was too small to make accurate measurements possible.

The object of the work on selective reflection was the following:

According to modern conceptions, selective reflection from insulators is occasioned by particles with definite free periods which are capable of vibrating in resonance with certain incident radiations. As the phenomenon is looked upon as taking place entirely within the molecule, it was thought of interest to determine, first, whether the selective reflection of a substance was dependent on its physical state, and, secondly, whether the mechanism giving rise to this selective reflection was localized within some definite portion of the molecule. As to the first point, the selective reflection of a salt was studied, both when the salt was solid and when molten; and as to the second point, an investigation was carried out on the selective reflection of a number of salts having a certain radical in common.

Having found that a molten salt is capable of reflecting selectively, a number of other molten salts and liquids were investigated for

[1] *Pogg. Ann.*, Jubelband, 336, 1874.

similar effects. In the case of sulphuric acid some very remarkable changes in the reflection-curve were found as the acid was diluted. Each of these subjects in turn will be discussed in detail.

DESCRIPTION OF APPARATUS

Radiometer-spectrometer.—The general arrangement of the apparatus is shown in the following diagram (Fig. 1):

Radiations coming from a Nernst glower (N) placed in front of the slit (S_1) were rendered parallel by the concave mirror (M_1),

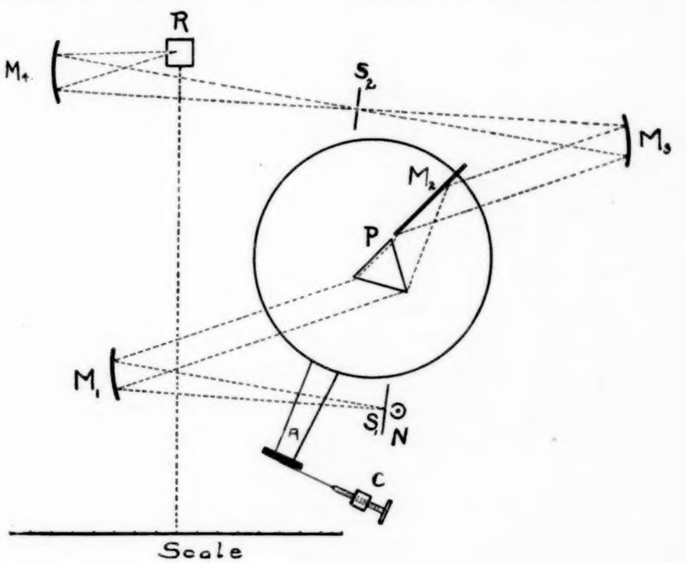


FIG. 1

and then passed through the Wadsworth mirror and prism arrangement (PM_2). The mirror (M_3) brought the spectrum of the source to a focus on the second slit (S_2). Thus, by a rotation of the prism and mirror system (PM_2) mounted on the spectrometer, it was possible to cause the entire spectrum to pass over the slit S_2, and in this manner any desired portion could be brought to a focus on one of the blackened vanes of the radiometer R. The slits S_1 and S_2 were usually opened to a width of 0.5 mm.

As mentioned, the source of radiation was a Nernst glower inclosed in a brass cylinder which had but one narrow opening along its

side. In this manner the disturbing effects of air currents were removed, as shown by actual tests of the constancy of the radiation of the glower. The concave mirrors (M_1) and (M_3) had a radius of curvature of 52 cm, while the mirror M_4, which concentrated the radiations on the radiometer vane, had a radius of curvature of only 26 cm. The rock-salt prism (P) was about 5 cm on an edge and had a refracting angle of 60° 10′ 6″. The calibration was carried out in the usual manner of calculating from the known dispersion-curve of rock salt, the deviations corresponding to definite wavelengths. To check the results thus obtained, the position of the emission bands of CO_2 from a Bunsen burner (2.70 μ and 4.40 μ), and also the position of the bands of metallic reflection from quartz (8.49 μ and 9.03 μ)[1] and from Iceland spar (6.69 μ and 11.41 μ), were observed, and were found to lie exactly on the curve.

Concerning the spectrometer the only point worth mentioning is that the rotation of the prism and mirror system, which was mounted on the spectrometer table, was measured by means of a micrometer (c) connected with the projecting arm (A) through a piece of steel tape. The length of the arm was so chosen that one division on the divided barrel of the micrometer corresponded to 6 seconds of arc.

The radiometer was of the usual Nichols type and was supplied with a rock-salt window. The sensibility of the instrument could easily be varied by changing the length of the quartz fiber which was partly wound up on a miniature reel arrangement. In the polarization experiments the sensibility was such that a meter-candle gave rise to a deflection of about 1000 mm, while in the later reflection experiments the sensibility was reduced to six-tenths of this value. With the exception of the rock-salt window and the minute concave mirrors about to be described, the radiometer was identical with the one employed by J. T. Porter[2] and described by him.

Throughout the course of the entire investigation this radiometer-spectrometer remained unmodified, all of the special apparatus being brought out in front of the slit S_1.

Minute concave mirrors.—Undoubtedly the easiest way of deter-

[1] These values, obtained by Rosenthal (*Ann. der. Phys.*, **68**, 783, 1899), are probably the most accurate known.

[2] *Astrophysical Journal*, **22**, 229, 1905.

mining the deflection of a radiometer is to observe the motion of the image of an incandescent lamp filament projected on a ground glass scale by means of a small concave mirror. Heretofore this method has not been used, for the reason that small concave mirrors weighing a milligram or two have not been obtainable. It occurred to me that they might very easily be made in the following manner.

A small concave mirror or good spectacle lens of about 1 m radius of curvature is heavily silvered and polished on its concave side, and is then cut up into strips about 4 mm wide. If, now, one of these strips be struck a smart tap on its edge by means of a flat file, small scales of glass bearing silver on one side can be caused to fly off. These scales are, of course, the concave mirrors in question. With a little practice it is easy to produce mirrors of this kind varying from 7 or 8 mm in diameter down to infinitesimal dimensions. These mirrors are found to retain their figure well, and to give images of surprising sharpness and brilliancy.

In making such small mirrors care must be taken to select a concave mirror of good figure, and an easy way to carry out a test is to diaphragm off all of the mirror but an area about 2 mm square, and then to examine the image produced. If it be necessary to use these mirrors in the open air, where silver will tarnish, it would be advisable to platinize the reflecting surface by cathode discharge.

POLARIZATION IN THE INFRA-RED

Reflection from glass and selenium.—Ultra-violet, visible, and short infra-red rays may readily be polarized by means of some such device as a Nicol or Rochon prism. However, beyond 2.5 μ in the infra-red spectrum this method fails on account of the opacity of the doubly refracting substance, Iceland spar, used in the construction of the above-named polarizers. Probably the most convenient method of producing polarized radiations in the regions of great wave-lengths is that involving reflection at the polarizing angle from some transparent substance. The conditions which such a substance must fulfil are:

1. It must have a fairly high reflecting power.

2. Its polarizing angle must be practically constant for different wave-lengths.

3. It must polarize very completely.

The obvious manner of attacking the problem is to make a study of the reflection-curves of various substances which appear to give promise of fulfilling the above-mentioned conditions. The apparatus used in carrying out such measurements is shown in the following diagram (Fig. 2).

Radiations from the Nernst glower at N were reflected at an angle of incidence of 10° from the two mirrors (M_2) and (M_3), and were finally brought to a focus on the slit S_1 of the spectroscope already described. The mirror whose reflecting power was to be determined

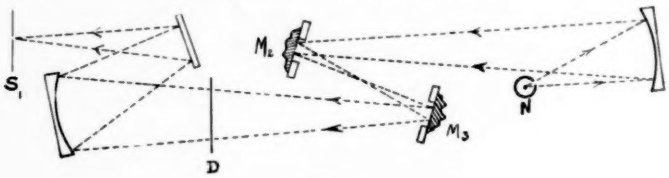


FIG. 2

was held in position at (M_3) by being pressed against a brass plate into which a hole had been cut. Any desired mirror could be placed behind the opening of a similar brass plate at (M_2). In order to keep the deflections on the scale, a diaphragm was placed at D which could be made to cut down the vertical aperture of the beam. Measurements of reflecting power were obtained in the following manner. The mirror to be tested was placed in position at M_3, and the radiometer deflection was noted; then a silver mirror was made to replace the former, and again the deflection was noted. The ratio of the latter deflection to the former gave the reflecting power of the substance for the wave-length corresponding to a given spectrometer setting.

Considerable time was lost in trying to adjust the two mirrors (i. e., the one of silver and the other of the material under investigation) upon a rotating table which could be made to move between stops. Due to the fact that the replacement of one mirror by another was not absolutely exact, very discordant results were obtained. Consequently this method was abandoned, and the one already described was used—thereby making the replacement of one mirror by another exact beyond a question. As might be expected, very concordant results were obtained.

In looking over some unpublished results on the reflecting power of amorphous selenium, it appeared to me that the reflecting power of this substance gave promise of becoming constant for the longer wave-lengths, and it was decided to make some exact measurements. Furthermore, since glass has been repeatedly used in the infra-red as a polarizer, it was thought worthy of interest to determine in what regions of the spectrum such a procedure would be permissible. The results as plotted in the following curves (Fig. 3) speak for themselves.

That glass would be unfit to act as a polarizer throughout the entire spectrum might have been predicted from a knowledge of

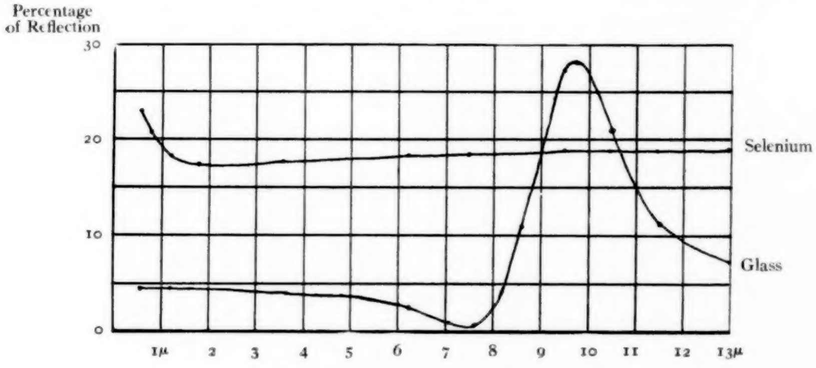


FIG. 3.—Glass and Selenium.

the fact that it contains SiO_2, which gives rise to the bands of metallic reflection of quartz. In the case of selenium it will be observed that the reflecting power is not only very high, but also very constant; hence this substance is eminently adapted to serve as a polarizer.

Since in experiments with polarized light two reflections are necessary, one from the polarizer and one from the analyzer, the maximum attainable energy after two reflections is to the incident energy as $r^2:1$, where r is the reflecting power of the polarizing medium. This statement would be rigorously true if the incident energy were already polarized in the plane of incidence. Taking into account the fact that this is not the case—i. e., considering the fact that the incident radiations are unpolarized—calculations have been made upon the basis of Fresnel's formulæ to show how great

the gain in energy is when selenium is used as polarizer and analyzer in place of other substances.

Refractive Index of Substance	Fraction Refl. from Polarizer	Fraction Refl. from Analyzer	Ratio of Emergent to Incident Energy	Gain in Using Selenium
2.565 (selenium)	0.27	0.54	0.146	
1.60	.095	.19	.018	8.1 times
1.50 (rock-salt)	.0745	.149	.011	13.3 "
1.40 (fluorite)	.0515	.103	.0053	27.6 "
1.30 (fluorite)	.0375	.075	.0028	52.0 "

Since selenium is comparatively transparent in the infra-red, it will be permissible to apply Fresnel's reflection formulæ and calculate the refractive index (n):

$$n = \frac{1+\sqrt{r}}{1-\sqrt{r}} .$$

Near $13\,\mu$, where the reflection-curve becomes flat, $r=19$ per cent., and the refractive index as calculated is $n=2.565$. Now, according to Maxwell's equations the relation $n^2=\epsilon$ (where ϵ is the dielectric constant) is fulfilled if the measurements be made sufficiently far removed from the region of absorption bands. Recently Schmidt[1] has found that for amorphous selenium $\epsilon=6.60$, while from the foregoing measurements we have $\epsilon=n^2=6.58$. This shows that the Maxwellian relation is already fulfilled, and it seems only reasonable to suppose that the reflecting power, and hence the polarizing angle, will remain constant throughout the remainder of the infra-red spectrum.

The selenium mirrors used in the experiments about to be described were prepared in the following manner. Some pure selenium was melted down in a porcelain crucible and was poured on a plate of hot glass; a second plate of hot glass was then quickly pressed down upon the pool of molten selenium, and the whole was laid on an iron plate to cool. The layer of selenium, after being pressed out, was usually about 1 mm in thickness. Heretofore considerable difficulty was experienced in getting the second plate to come off. I have found, however, that this can be accomplished very easily

[1] *Ann. d. Phys.*, **2**, 114, 1903.

if the second plate be made considerably longer than the first, thereby making it possible to bend it slightly and thus cause it to tear itself away from the selenium. Very beautiful mirrors may be obtained in this manner. Those actually used were of oval shape, about 10 cm long and 5 cm wide, and had a figure as perfect as that of the glass plate which had covered them.

By means of such mirrors as these a polarizer of the following form was constructed (Fig. 4):

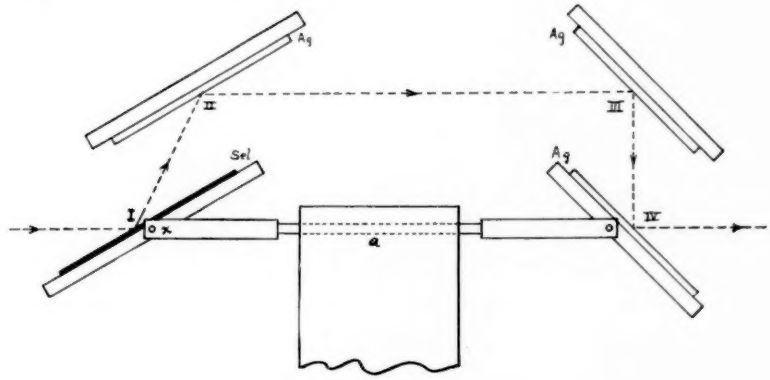


FIG. 4

Selenium polarizer and analyzer.—The selenium mirror (1) and the silver mirror (2) were adjusted parallel, and then rigidly connected with one another, so that a rotation about the axis (x), changing the angle of incidence, would not change the direction of the emergent beam. The two other silver mirrors (3) and (4) were also adjusted parallel to one another, and the entire system could be rotated about the axis a (the axis a being perpendicular to the axis x). It is easy to see that in an apparatus of this kind the incident and emergent beams lie along the same straight line, and a rotation about the axis a will not affect the direction of the emergent beam. Therefore, if this beam be concentrated upon the slit of a spectrometer, it will remain there in spite of the rotation of the mirror system.

In the actual experiments the instrument just described was used as the analyzer, while the polarizer was made to consist of the mirrors (1) and (2) alone. In order to measure the rotations of the system of mirrors, small protractors graduated to 30′ were fas-

tened to the axis a. By means of a telescope tenths could be easily estimated, thus making it possible to read to $3'$ of arc.

Complete polarization of infra-red radiations.—As was mentioned in the beginning, the first question taken up was whether this apparatus would actually polarize infra-red radiations. The manner in which the experiment was carried out is shown in the accompanying diagram (Fig. 5). The rays from the Nernst glower (N), after being rendered parallel by the concave mirror M, were polarized at P, and then analyzed at A. The concave mirror (M_2) focused

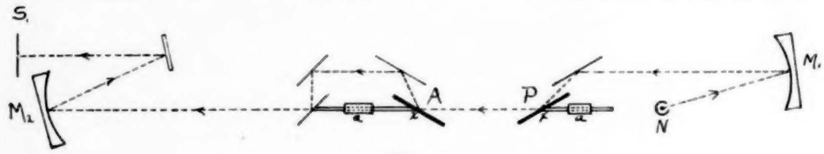


FIG. 5

the image of the Nernst glower on the slit S_1 of the spectrometer already described. Having roughly adjusted the selenium mirrors of the polarizer and analyzer in the position of maximum polarization, they were "crossed," and the position of least reflection was determined by successive rotations of the systems about the axes x and a. This position could be found very easily, and the fact was established that variations in the angle of incidence of a degree or two did not markedly influence the results. Deflections of the radiometer were observed up to 13 μ corresponding to the condition of parallel and crossed polarizer and analyzer. Results of the following character were obtained throughout this entire range of wave-lengths:

Relative Position of Polarizer and Analyzer	Radiometer Deflection
Parallel	> 1000 mm
Crossed	< 1 mm

This shows, then, that the radiations are polarized to a very high high degree. Of course, no claim is made to complete polarization, as it is found that, if the incident energy be increased very greatly, a small deflection of a few divisions is obtained. All things taken into consideration, however, selenium is found to fulfil the conditions imposed upon it as a polarizer very well indeed.

Elliptical polarization from Iceland spar in the region of metallic

reflection.—In order to determine whether a non-metallic substance in its region of metallic reflection behaves as a metal toward polarized light, a parallel beam of radiations (Fig. 6) from a Nernst glower (N) polarized in an azimuth of 45° at P was caused to be reflected from a surface of Iceland spar (I) (polished on one of its

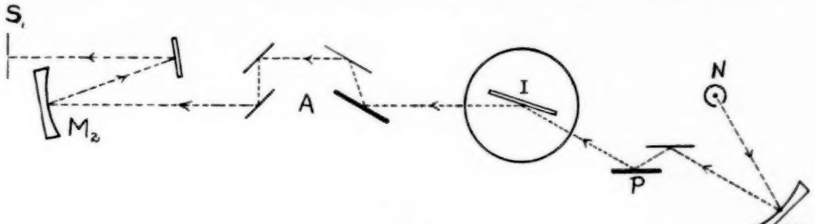


FIG. 6

cleavage planes) at an angle of incidence of 65°. After passing through the analyzer (A), these radiations eventually reached the slit (S_1) of the spectrometer. For a given setting of the spectrometer, radiometer reflections were observed corresponding to the two positions of the analyzer giving maximum and minimum energy. The readings here recorded are for $\lambda=4\ \mu$, where Iceland spar reflects vitreously, and for $\lambda=6.70\ \mu$, where it reflects metallically.

	Maximum Deflection	Minimum Deflection	Ellipticity (Ratio of Axes)
$4.0\ \mu$........................	501 div.	0 div.	10:0
$6.7\ \mu$........................	90 "	55 "	10:6

These results show conclusively that Iceland spar transforms plane into elliptically polarized light within the region of metallic reflection.

On account of the difficulty of preparing plates of Iceland spar or quartz sufficiently thin to make transmission and absorption measurements possible within the regions of metallic reflection, it was thought feasible to make these measurements by an indirect, katoptric method. As stated earlier in the paper, the amount of energy reaching the radiometer was found insufficient to make such determinations possible. When it is considered that the radiations suffered fifteen reflections before reaching the radiometer, it is not surprising that the available amount of energy was found too small.

However, the work of Pflueger (*loc. cit.*) and Nichols[1] on insulators, and of Hagen and Rubens[2] and Minor on metals, has unquestionably established the fact that strong absorption and metallic reflection go together. In view of the fact that any explanation of metallic reflection involving the conception of resonance is not applicable to metals, one is led to suspect that metallic reflection, wherever found, is, in all probability, to be attributed to strong absorption alone.

SELECTIVE REFLECTION IN THE INFRA-RED

Reflection from solid and molten salt.—In order to determine the selective reflection of various substances, such investigators as Rubens and Nichols,[3] Aschkinass,[4] and Porter (*loc. cit.*) made use of the method of multiple reflections, commonly known as the method of "Reststrahlen." On account of the weak reflecting power of most of the substances investigated in the present work, it was decided not to use the above method, but rather to make direct determinations of the reflecting power. In the case of solids the same method was used as that described in the chapter dealing with the reflection from selenium, while in the case of liquids the method used involved the reflection of energy from the free liquid surface.

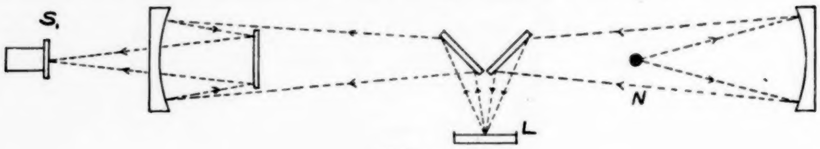


FIG. 7

The general arrangement of apparatus is shown in elevation in the diagram (Fig. 7):

Radiations from the Nernst glower N were first brought to a focus on the surface of the liquid at L, and finally on the slit S_1 of the spectrometer. Measurements were obtained in the following manner: With the liquid surface in a fixed position a complete series of readings was taken throughout the entire spectrum; then the liquid surface was replaced by one of silver, and a similar series

[1] *Sitzungsberichte der Kgl. Preuss. Akad. der Wissenschaften*, **44**, 1, 1896.

[2] *Ann. d. Phys.*, **8**, 1, 1902.

[3] *Ibid.*, **60**, 418, 430, 1897.

[4] *Ibid.*, **65**, 241, 1898.

was taken. By proceeding in this manner it was possible to make corrections for a rising or falling energy-curve of the source. The principal reason for adopting this method of procedure in preference to the one used before was that certain liquids, especially the molten salts, had a most annoying tendency to form "surface skins," and it was only by stirring the molten mass vigorously, and then taking the readings in the shortest time possible, that concordant readings could be obtained. It might be added that the radiations sent out by the Nernst glower were of an amply sufficient constancy to make

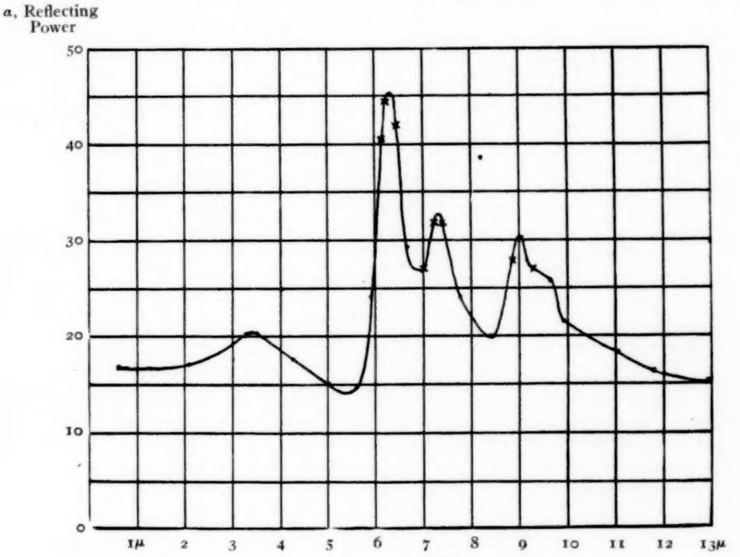


FIG. 8.—Sodium-Potassium Tartrate.

such a procedure allowable. On each of the curves plotted it is indicated whether the ordinates are proportional or equal to the reflecting power.

The intensity of reflection from most of the liquids examined, even at the maxima, lies under 15 per cent. The only exception is sulphuric acid, which at its highest maximum reflects 20.8 per cent.

The first point to be taken up was to show that, so long as the molecule as a whole remains unaltered and no changes take place in the nature of the surrounding medium, the position of the bands of metallic reflection will remain the same. The substance chosen

to show this was sodium potassium tartrate. A reflection-curve was obtained from a polished crystal, and then an attempt was made to obtain a similar curve for the molten salt. Unfortunately,

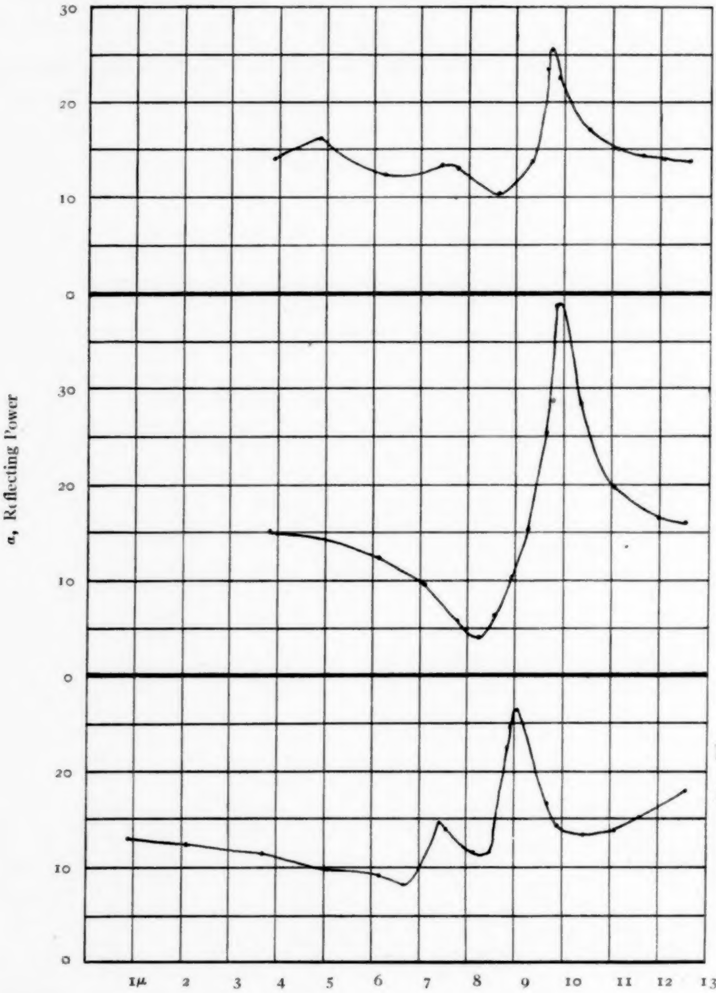


FIG. 9.—Glycerine.

FIG. 10.—Sodium Silicate.

Fig. 11.—Nitroso-Dimethyl-aniline.

however, it was not possible to obtain a complete curve of this kind, on account of the rapid formation of a surface skin, and in consequence I had to content myself with a determination of the positions of the maxima, which were very marked. This was done by setting

the spectrometer approximately in the position of a band, then removing the surface skin and making an accurate setting before a new skin had time to form. The readings thus obtained are indicated by crosses on Fig. 8, and show that the position of the bands remain unaltered.

Reflection from liquids.—In view of the fact that molten sodium potassium tartrate was found to possess bands of metallic reflection,

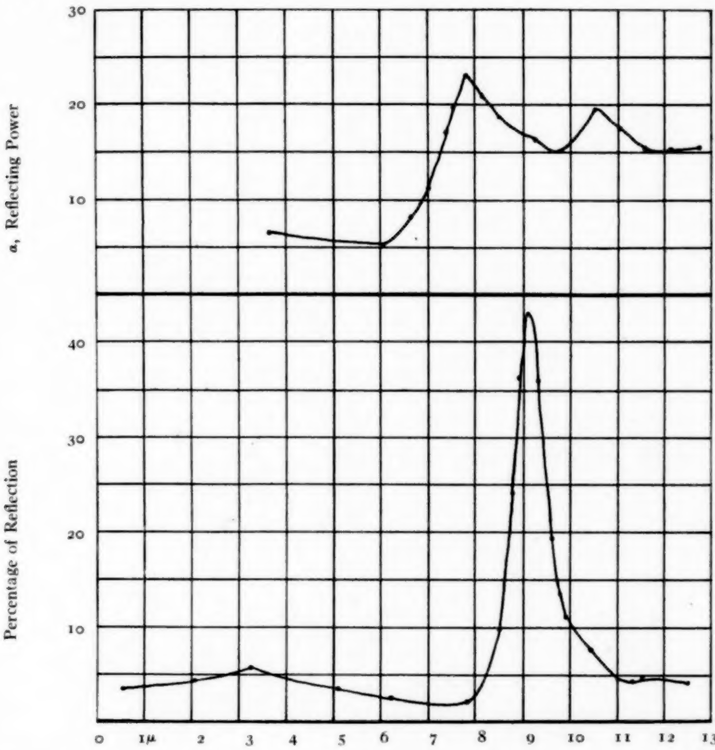


FIG. 12.—Nitric Acid.

FIG. 13.—Nickel Sulphate.

it was deemed of interest to investigate other molten salts and liquids for similar effects. The accompanying curves (Figs. 9–12, 14, and 15) for glycerin, liquid sodium silicate, molten nitroso-dimethyl-aniline, nitrates of calcium, cobalt, magnesium and ammonium, nitric acid, and sulphuric acid show in a most striking manner that selective reflection is by no means confined to solids.[1] It might be added

[1] No marked bands of selective reflection were found for wave-lengths shorter than 3 μ.

that no such marked maxima were obtained when water, alcohol, molten caustic potash, phosphoric acid, acetic acid, and hydrochloric acid were tried.

Localization of mechanism of selective reflection within molecule.— Upon looking over the results obtained, it became very evident that

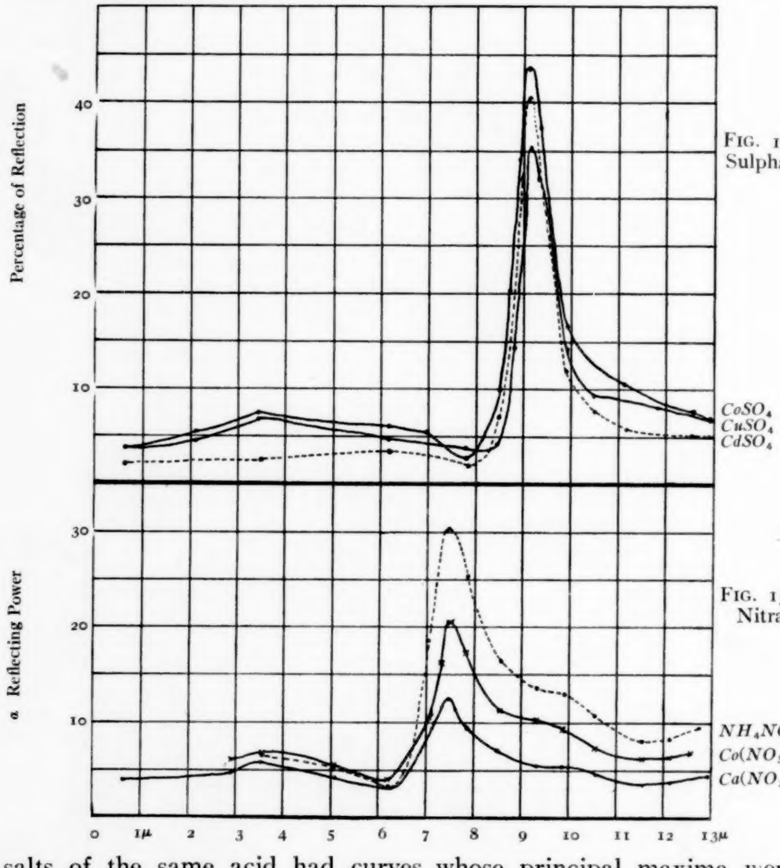


FIG. 14.— Sulphates.

FIG. 15.— Nitrates.

salts of the same acid had curves whose principal maxima were situated in the same region of the spectrum, and it was decided to investigate this point more fully. Consequently, the selective reflection of the following substances was investigated: sulphates of copper, nickel, cadmium, iron, sodium, potassium, and cobalt; also the nitrates of silver and potassium in addition to those of calcium, ammonium, magnesium, and cobalt already investigated. Whenever

crystals were obtained whose surfaces were of sufficient size and perfection, the absolute values of the reflecting powers were determined. Where this was not the case, it was possible to obtain only

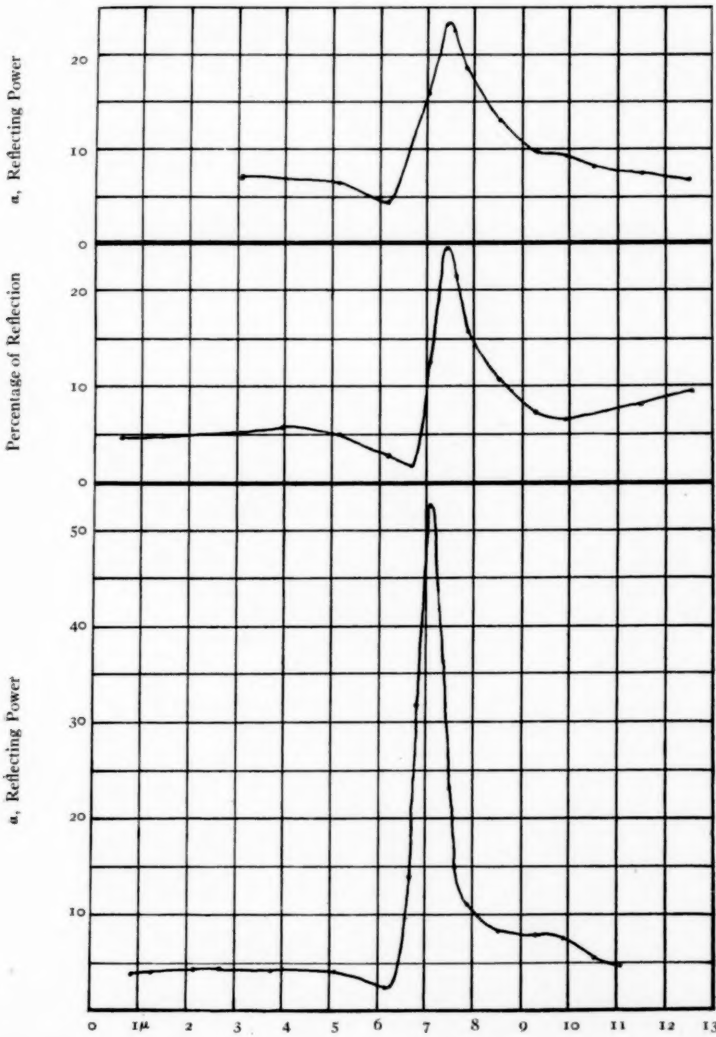


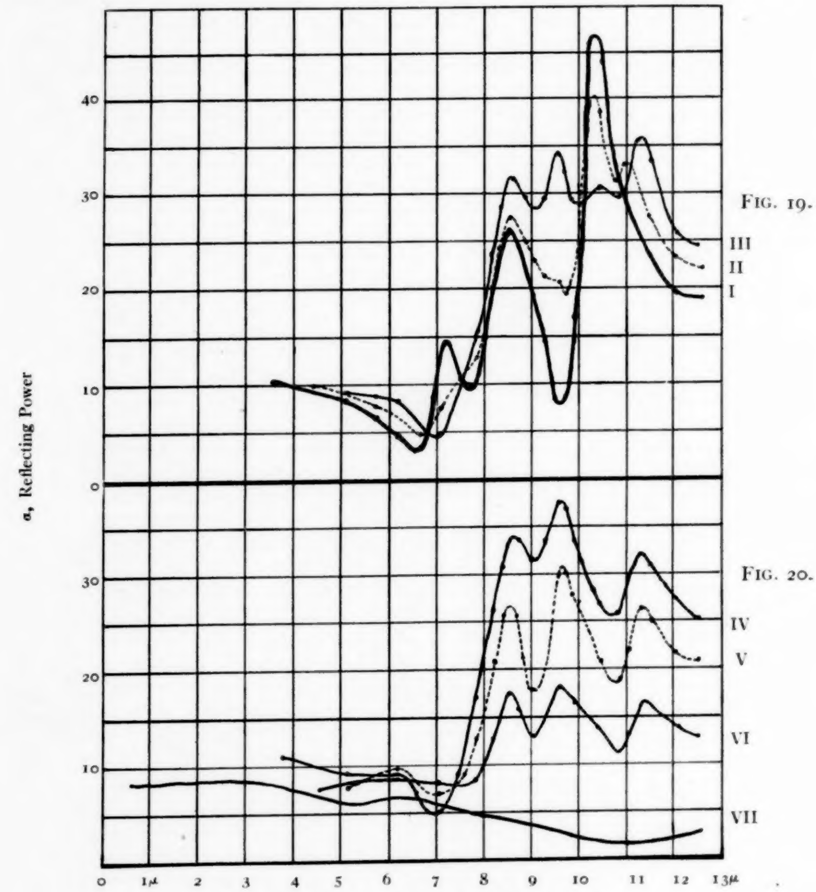
FIG. 16.—Magnesium Nitrate.

FIG. 17.—Silver Nitrate.

FIG. 18.—Potassium Nitrate.

relative values of the reflecting power. Finally, in the case of the sulphates of sodium, potassium, and iron only a determination of the positions of the maxima was made.

An inspection of the following curves, as for example those of the sulphates (curves 13 and 14), cannot fail to impress one with the fact that the maximum near 9.05 μ is present under all circumstances, no matter what the nature of the metal in the molecule may



FIGS. 19 and 20.—Fuming Sulphuric Acid.

be. Again in the case of the nitrates, it is equally evident that the strong maximum near 7.40 μ appears in all cases. Attention must, however, be called to the fact that the position of the maxima for the salts of a given acid are not identically the same as will be observed if the curves for silver nitrate and potassium nitrate (Figs. 17 and

18) are compared; nor do the maxima of sulphuric and nitric acid coincide with those of the sulphates and nitrates respectively. These points will be discussed later on. However all this may be, one cannot help but acknowledge that the similarity of the curves, especially of those shown in Figs. 14 and 15, is too striking to be without significance. Since every salt of a given acid thus far studied shows a maximum in approximately the same region of the spectrum, in spite of marked changes in the nature of the metal, one is strongly tempted to localize the mechanism giving rise to this maximum in the acid radical of the molecule, i. e., that portion of the molecule which in solution becomes the negative ion.

Reflection from fuming sulphuric acid at various dilutions.—In order to reassure myself of the correctness of the results which had thus far been obtained, a second series of measurements was carried out for all of the substances studied. All of the results were verified, with the exception of those for sulphuric acid, which were totally different from the first. As was later found out, this was due to the fact that the wrong sulphuric acid bottle (containing weak acid) had been used. On account of the striking difference in the two curves, it was decided to make a systematic investigation of the changes in the forms of the curves corresponding to different degrees of dilution of the acid. The curves Figs. 19 and 20 represent the results obtained.

Curve	I	is for	undiluted fuming sulphuric acid of density 1.87			
"	II	" "	12 parts	of this acid and	1	part of water
"	III	" "	12 "	" " " "	2.4	" " "
"	IV	" "	3 "	" " " "	1	" " "
"	V	" "	1 part	" " " "	1	" " "
"	VI	" "	1 "	" " " "	3 parts	" "
"	VII	" "	pure water (distilled).			

These proportions of acid to water are given in terms of volumes.

In curve I three distinct maxima are discernible—at 7.20 μ, 8.65 μ, and 10.30 μ; and also three minima—at 6.62 μ, 7.73 μ, and 9.64 μ. Curves II and III, which represent intermediate stages, show that the maximum at 10.30 μ is rapidly disappearing; the minimum at 9.46 μ is being filled in; the maximum at 8.65 μ remains, while the maximum at 7.20 μ also disappears. In addition to these

changes, other maxima are coming in, whose positions from curves IV and VI are seen to be at 9.60 μ and 7.40 μ. As will be observed, these maxima are distinctly new and are not the old ones displaced to another position. Furthermore, it is evident from curve VII that water superimposes no peculiarities of its own on the curves for sulphuric acid.

I have tried to interpret these results in the following manner.

It is generally conceded that in sulphuric acid of the greatest strength used in these experiments there is a large excess of undissociated H_2SO_4 molecules. By an addition of water, these molecules are first broken down into $\overset{+}{H}\ \overset{-}{HSO_4}$ ions, and finally into $\overset{+}{H}\ \overset{+}{H}\ \overset{-}{SO_4}$ ions—the complete change to the latter form taking place only at infinite dilution. Considering the fact that these changes in the molecules and ions are accompanied by marked changes in the reflection-curves, it seems only reasonable to suppose that those maxima, which are at first present and then disappear with increasing dilution, are due to molecules or ions (or possibly both) which also disappear with increasing dilution. Similarly, the new maxima which appear are supposed to be due to new ions which are formed in consequence of increasing dilution. It would, indeed, be premature to attempt to attribute with definiteness certain reflection maxima to certain molecules or ions; for the state of our knowledge as to the condition of affairs existing in solutions is as yet too incomplete. In the present case, for example, we have fuming sulphuric acid (which is a solution of SO_3 in H_2SO_4) and water. Not only is it possible to have a large number of different kinds of ions, but in addition we may have a large number of complexes or hydrates which the molecules of the acid might form with those of the water. From this it will be seen that much work remains to be done to clear up the subject.

But let us return once more to the work on the sulphates and nitrates. As already pointed out, this work has made it seem probable that the mechanism giving rise to certain strong maxima of reflection is localized within the acid radical. Now, this conclusion is quite in accord with the results of some calculations which Drude[1]

[1] *Annalen der Physik*, **14**, 677, 1904.

has recently carried out. In these it is shown that the particle which, in consequence of its sympathetic vibrations, gives rise to the ultra-violet absorption band has a charge and a mass identical with that of the corpuscle, while the particle giving rise to the infra-red absorption band has a mass of the order of magnitude of the molecule itself. Since it is a large portion of the molecule which is looked upon as being thrown into vibration, it does not seem difficult to account for the fact that the positions of the reflexion maxima for the salts of the same acid are not the same. There is but little doubt in my mind that the positions would be the same if the acid radicals could execute their vibrations independently of other particles surrounding them—a condition approached at infinite dilution. In the cases actually studied the freedom of motion of the acid radical depended not only on the closeness of the bond between it and the metal ion, but also upon molecules of water clustering around it. It does not seem unreasonable, therefore, to suppose that these influences might affect the period of vibration of the acid radical, and hence the position of the reflection maximum.

The following table gives a list of the substances studied, together with the positions of the principal maxima of reflection:

Substance	Formula	Position of Reflection Maxima			Remarks
Na-K tartrate	$C_8H_4KNaO_6 \cdot 4H_2O$	6.35	7.35	9.05 μ	Solid crystal
Magnesium nitrate	$Mg(NO_3)_2 \cdot 6H_2O$	7.45			Molten
Cobalt nitrate	$Co(NO_3)_2 \cdot 6H_2O$	7.45			"
Ammonium nitrate	NH_4NO_3	7.45			"
Calcium nitrate	$Ca(NO_3)_2 \cdot 4H_2O$	7.45			"
Silver nitrate	$AgNO_3$	7.45			Solid crystal
Potassium nitrate	KNO_3	7.05			" "
Nickel sulphate	$NiSO_4 \cdot 7H_2O$	9.05			" "
Cobalt sulphate	$CoSO_4 \cdot 7H_2O$	9.05			" "
Copper sulphate	$CuSO_4 \cdot 5H_2O$	9.15			" "
Cadmium sulphate	$CdSO_4 \cdot 4H_2O$	9.10			" "
Ferric sulphate	$Fe_2(SO_4)_3 \cdot 9H_2O$	9.05			" "
Sodium sulphate	$Na_2SO_4 \cdot 10H_2O$	9.02			" "
Potassium sulphate	K_2SO_4	8.85			" "
Fuming sulphuric	H_2SO_4 and SO_3	7.20	8.60	10.35 μ	Naturally fluid
Acid and water	in H_2O	8.60	9.60	11.35	" "
Nitric acid	HNO_3	7.85	10.55		" "
Glycerine	$C_3H_8O_3$	4.80	9.70		" "
Na-Silicate	Na_2SiO_3	9.95			" "
Nitroso-dimethyl aniline	$(CH_3)_2NC_6H_4NO$	7.40	9.00		Molten

SUMMARY

The results of this investigation may be summed up briefly as follows:

1. The reflecting power of amorphous selenium has been studied out to 13 μ. In consequence of the high and constant reflecting power of this substance, it was used in the construction of a polarizer and analyzer, adapted for work throughout the entire infra-red spectrum.

2. It was shown that infra-red radiations are capable of being polarized out to a wave-length of 13 μ, which was as far as the experiments could be carried.

3. It was shown that the non-metallic substance, Iceland spar, in its region of metallic reflection transforms plane into elliptically polarized light by reflection. This shows that, so far as its behavior toward plane polarized light is concerned, there is nothing to distinguish a non-metal from a metal.

4. Upon finding that the positions of the bands of selective reflection from a solid salt (*Na-K* tartrate) remain unchanged when the salt is molten, it is concluded that the mechanism giving rise to these bands is not affected by the freedom of motion of the molecule as a whole, and is therefore in all probability localized within the molecule itself.

5. By examining numerous liquids, it was found that these, as well as the solids, possess bands of selective reflection in the infra-red.

6. In the case of fuming sulphuric acid it was found that marked changes in the reflection-curves appeared when the acid was diluted. It was concluded that these changes were due to the breaking-down of certain compounds in solution and the consequent formation of new ones.

7. From the marked similarity in appearance and position of reflection maxima of the salts of a given acid (nitrates and sulphates) it was concluded that the mechanism giving rise to these maxima was localized within the acid radical.

Most of the substances used in this work were obtained through the kindness of Professor Harry C. Jones, of the Chemical Labora-

tory, and it is with pleasure that I acknowledge my indebtedness to him.

The present investigation has been carried out during the past year under the direction of Professor Ames, whom I wish to thank most heartily for his many valuable suggestions and his never-failing interest in the progress of the work.

JOHNS HOPKINS UNIVERSITY,
April, 1906.

ON A NEW FORM OF SPECTROHELIOGRAPH

By G. MILLOCHAU and M. STEFÁNIK

In presenting a new form of spectroheliograph, we consider it advantageous to recall the principal points in the history of this instrument.

In 1869 M. Janssen[1] described an apparatus for the observation of monochromatic images of luminous objects; this apparatus still exists at the Meudon Observatory, and we reproduce a photograph of it herewith (Fig. 1). It consists of a direct-vision spectroscope, in which the collimating lens is movable between two screws, permitting the spectrum to be displaced slightly. At the focus of the telescope lens is a second slit, the two jaws of which can be independently adjusted and used to isolate the desired radiation. This slit is observed with a positive eyepiece. The spectroscope thus described is contained within a tube, which can be moved rapidly about its axis by means of a system of gears. This instrument thus constitutes a spectrohelioscope, and was intended for the visual study of the prominences; but by substituting a sensitive plate for the eyepiece it might be immediately transformed into a spectroheliograph. M. Janssen's apparatus embodies the principal characteristics of the spectroheliograph, although his idea did not receive practical application during a score of years.

FIG. 1

[1] *C. R.*, **68**, 94, 713, 1869. *Ibid.*, *Inst.*, XXXVII, p. 397, 1869. *Mondes*, (2) XXI, p. 420.

During this period Braun and Lohse devoted themselves to the question. Braun, director of the Kalocsa Observatory, described a spectroheliograph in the *Astronomische Nachrichten* in 1873. His idea was to cause a two-slit spectroscope to move about a pivot passing through the point of intersection of the axes of the collimator and telescope. The image of the second slit was to be photographed by means of a camera. This apparatus was to be attached to an equatorial and moved by a mechanical device bearing upon a fixed point. Unfortunately, this instrument was not constructed, because of a lack of funds, and the idea remained unknown. Subsequently Lohse, of Potsdam, made unsuccessful experiments with a spectroheliograph of his invention.

Hale, who was not then acquainted with the investigations of his predecessors, devised two types of spectroheliographs and described them in 1889. In one of these devices the solar image was allowed to move across the first slit, while the photographic plate, placed immediately behind the second slit, was moved simultaneously; in the other plan the two slits were movable and the other parts of the apparatus stationary. In 1891 he obtained, with an instrument of the second type, the first photographs of the chromosphere and prominences, and in 1892 he extended his method to the study of the entire disk of the Sun.

A little later Mr. Evershed mounted his spectroheliograph (with direct-vision prism), which may be regarded as a special case of Braun's.

At the end of 1893 M. Deslandres published in the *Comptes Rendus* of the Paris Academy of Sciences the first results obtained with his spectroheliograph. His apparatus consists of a one-prism spectroscope, of small dispersion, placed on a carriage, by means of which it can be moved in a horizontal direction, perpendicular to the axis of the collimator. The photographic plate is moved behind the second slit by a system of levers attached to a fixed point.

A general defect of spectroheliographs is the tendency to record on the photographs all the vibrations produced by the various rolling or sliding parts comprised in their construction. This defect is due to the very principle of the instrument, according to which the solar image is built up by the integration of a line. Hence, in order

to obtain the best possible results, it is necessary to construct these instruments with special care, which renders them very costly and also difficult to operate.

In the arrangement which we have the honor to propose, we believe we have reduced the rolling and sliding friction to a minimum, and consequently have provided a means of diminishing, in large measure, the difficulty just mentioned. A two-slit spectrograph, of any form, is mounted so as to move around a horizontal axis perpendicular to the plane containing the optical axes of the spectrograph (Fig. 2). This arrangement is realized by the use of an axis turning between two centers. The motion is produced by a Brashear clepsydra, mounted vertically. It is connected with the spectro-

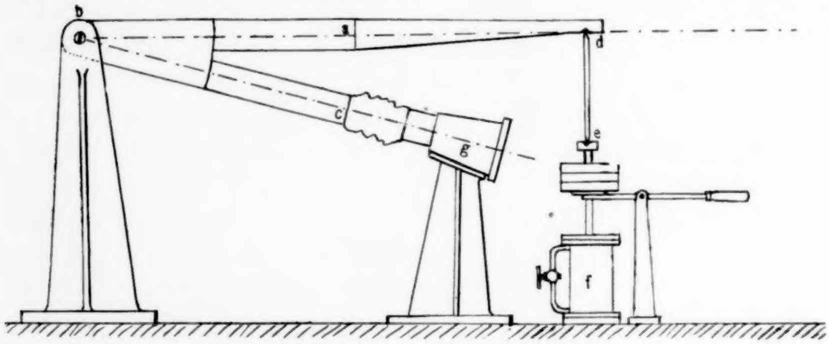


FIG. 2.—Diagram of Spectroheliograph.

a, *b*, *c*, spectrograph with grating.
b, horizontal axis of rotation.
a, first slit; *c*, second slit.
d, *e*, stem of junction with Brashear clepsydra, *f*.
g, fixed photographic apparatus for securing image given by movement of second slit.

graph by a bar with pointed extremities, which enter two conical holes, one of which is on the spectrograph in the prolongation of the optical axis of the collimator, while the other is at the end of the piston-rod of the clepsydra. The axis of rotation of the spectroheliograph must pass through the point of intersection of the axis of the collimator and that of the telescope of the spectrograph. The distances between this axis and the two slits should be in the ratio of the focal lengths of the collimator and telescope objectives.[1]

[1] These two principles were pointed out by Braun in 1873 (*Astronomische Nachrichten*, **80**, 33–41, 1873).

In case a grating is employed,[1] the second slit may be stationary, and placed in the axis of the telescope; the setting on the spectral line can be accomplished by a slow rotation of the grating.

At its two extremities the slit is widened for the purpose of taking a photograph of a portion of the spectrum of the diffuse light of the sky, thus providing a simple means of determining the exact radiation with which the monochromatic photograph was obtained. A photographic plate may be placed immediately behind the second slit, supported by the stationary part of the instrument (Hale's arrangement), or the image may be photographed with a separate camera (Braun's arrangement).

This spectroheliograph may receive light from a siderostat or a cœlostat, or it may be attached directly to an equatorial. In the last case its mounting should be capable of rotation about the optical axis of the objective of the equatorial, in order to render the plane of the spectrograph nearly vertical, and the Brashear clepsydra should be arranged so as to be placed nearly vertical for any position of the Sun.

For solar investigations the invention of the spectroheliograph has the same importance as the discovery of the telescope in astronomy. It is a true monochromatic telescope, without which certain details of the solar constitution would perhaps have remained unknown. The thanks of the scientific world are due to M. Janssen, who devised and realized the first spectrohelioscope, and to Mr. Hale, who, though unacquainted with the earlier work, succeeded in constructing the first practical spectroheliograph and in obtaining the first monochromatic photographs of the Sun.

Observatoire de Meudon,
April, 1906.

[1] The recent discovery by Hale of the dark hydrogen flocculi has shown the advantage of employing a grating for this class of investigation.

Minor Contributions and Notes

PLANETARY INVERSION

In view of the discussion between Professor Moulton and Professor W. H. Pickering in the number of the *Astrophysical Journal* for December last, perhaps a brief abstract of the results obtained by applying Sir George Darwin's theory of tidal friction to the question of planetary inversion may be of interest.[1] It seems certain to be the case that if a planet unattended by any satellites has an initial retrograde rotation, its axis of rotation will, under the influence of tidal friction, tilt over until the planet reaches a position of stable equilibrium in which its rotation will be direct. The stable value for the obliquity will lie somewhere between 0° and 90°, its exact value depending on the planet's viscosity, rate of rotation around its axis and of revolution in its orbit.

If satellites are introduced, the question becomes more complicated, and the stable value of the obliquity will vary with the different conditions as to the number of the satellites, their masses, and mean distances. There will be three possible equilibrium values for the obliquity according to different circumstances—values very near 0°, 90°, and 180°, respectively. *Jupiter* is certainly approaching the first of these values; assuming for it initial retrograde rotation, its satellite system must have been evolved after its obliquity had under the influence of tidal friction alone decreased to some value less than 90°. *Saturn*, on the other hand, evolved *Phœbe*, and possibly also *Japetus* and *Hyperion*, while its obliquity was still greater than 90°. As its obliquity approached this value, *Phœbe's* orbit moved down to the ecliptic (thus remaining retrograde), while *Japetus* and *Hyperion* followed *Saturn's* equator over. Later on, the inner satellites were evolved, and under their influence and that of the ring *Saturn* is moving into a stable position of small obliquity. In the cases of *Uranus* and *Neptune*, lack of sufficient data makes it impossible to say with any accuracy what is happening but it seems most likely that *Neptune* is being driven by its one satellite into a stable position with an obliquity of 180°. The obliquity of *Uranus*, too, is possibly being increased at present, but in this case the result is very doubtful.

[1] The paper containing the details of the investigation, which was suggested to me by Professor H. H. Turner, has been published in the *Monthly Notices* of the Royal Astronomical Society, April, 1906.

As I have stated in my paper, the way in which Professor W. H. Pickering stated his theory led Professor Moulton to a not unnatural misconception as to the couple which would be responsible for the inversion of the planet. The misconception has in large part invalidated Professor Moulton's brief criticism of the theory and it underlies certain assumptions necessary for the validity of this criticism, but not, as I view the question, necessary for the validity of the theory. A careful examination of the objections raised by Professor Moulton has shown that they do not apply to the form of the theory which I have discussed.

It remains to add that the theory is beset by many difficulties, such as the great extent of time involved and the doubtful factor introduced by the heterogeneity of the planets. It does not, so far as I can see at present, explain the high obliquities of *Jupiter's* recently discovered satellites; but it is an hypothesis which does offer an explanation of the retrograde motion of *Phœbe* in its orbit, and of the retrograde rotations of *Uranus* and *Neptune*.

F. J. M. Stratton.

Caius College, Cambridge.

A MECHANICAL ILLUSTRATION OF THE PLANE GRATING

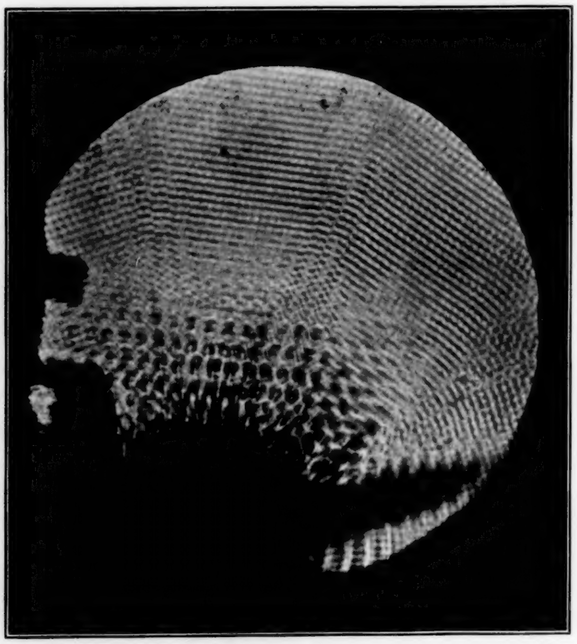
The experiment described below was devised for a semi-popular lecture to illustrate by mechanical waves the production of the various spectra by a plane grating. The scheme was to produce, on the surface of mercury, ripples emanating from a series of equidistant points distributed along a straight line. These points of course correspond to the spaces between the rulings of the grating, and the ripples to the secondary waves coming from these spaces.

In order to produce the ripples, a thin sheet of iron was cut into the form of a comb of sixteen teeth spaced 5 mm apart, and attached to the lower prong of an electrically driven tuning-fork, arranged to vibrate in a vertical plane. The frequency of the fork was not known, but was probably under 100. The apparatus was then arranged so that the teeth of the comb dipped into the surface of the mercury, which was contained in a large shallow, circular tray; the comb, however, was not set in the middle of the tray, but rather near one edge.

The waves set up when the fork was put in motion moved much too fast to be distinguishable by unaided vision, only the streaked appearance of the disturbed surface being visible in this way. However, when the surface was viewed through a stroboscope rotating at the proper speed, it was seen that close to the comb there was a chaotic mass of waves,

which farther out resolved themselves into several series of regular rectilinear wave-trains, advancing in different directions, and symmetrically distributed around a line normal to the comb at its middle point. These different series of wave-trains correspond respectively to the spectrum of order zero (moving out in a direction normal to the comb), and spectra of orders one and two on each side.

In order to project the waves on the wall for lecture purposes, use was made of a large cheap lens, of 20 cm diameter, and of 90 cm focal length. By means of this a considerable area of the mercury surface (a circle about



15 cm in diameter) was illuminated by *convergent* light from an arc, striking the surface with an angle of incidence of about 20°. Where the reflected light came to a focus (forming a clear image of the arc when the mercury was undisturbed), the projecting lens was placed to focus the image of the waves upon the wall, a good mirror being inserted in the path to deflect the rays in the proper direction. The scheme of focusing the light from the arc upon the projecting lens is of course essentially the principle of lantern-slide projection, and its great advantages, when applied to the projection of mercury waves, is sufficiently obvious. The

edges of the projected area are sharp and clear, and the illumination brilliant and practically uniform.

The figure is a reproduction of a photograph taken by placing a camera so that its lens replaced the projecting lens at the focus of the arc. The exposure was made by rapidly drawing across the path of the light an opaque screen containing a narrow slit; for the fastest speed on an ordinary commercial camera shutter, rated as $\frac{1}{100}$ second, was found to be too slow for the purpose.

Some trouble was caused by reflection from the walls of the tray, causing the formation of standing waves of small amplitude all over the surface. This was alleviated by lining the walls with corrugated strips of sheet iron, which to a certain extent dispersed the waves, instead of reflecting them regularly.

HERBERT M. REESE.

UNIVERSITY OF MISSOURI,
June 1906.

AN OCCULTING SHUTTER FOR CONCAVE GRATING SPECTROSCOPES

In almost any spectroscopic work with photography, if the plates are to be accurately measured, a comparison spectrum is a prime requisite; and in order to get this comparison spectrum on the photographic plate without any accidental shift with reference to the principal spectrum, some mechanical contrivance is usually necessary. For use with the concave grating, probably the best known and most efficient of these contrivances is the simple device due to Rowland. This consists of a flat bar, about one inch wide, one-eighth inch thick, and as long as the plate, having down its middle a slit of the same width as the thickness of the bar. It is mounted immediately in front of the plate, and arranged so that it can be turned through ninety degrees about its longest axis. When the plane of the flat face is vertical, only a strip down the middle of the plate is exposed to the grating; when it is horizontal, this central strip is covered, and the regions above and below are exposed.

This simple and convenient device has two difficulties. In order to secure room to turn, it cannot be placed very close to the plate, so that the edges of the exposed regions show a penumbral effect which sometimes becomes an annoyance. Moreover, the bar is mounted on the camera-box, and there is always the possibility that in turning it some slight jar will cause an appreciable movement of the plate lengthwise and so introduce between the lines on the two spectra a lateral shift which might be falsely ascribed to a difference in wave-length.

To obviate the first of these difficulties Hale replaced Rowland's rotating "shutter" by a pair of metal plates with suitable openings in them which are slid into grooves in the plate-holder close up to the surface of the photographic plate. First one of these plates is put in place and the central strip of the photographic plate is exposed. Then it is withdrawn, the other slid into its place, and the second exposure made, this time the central strip being covered and the edges exposed. Although this device successfully eliminates the penumbra effect, the chances for accidental displacements to occur are vastly greater than is the case with Rowland's simple shutter, so that it can hardly be recommended unless elaborate precautions be taken to detect such displacements.

In a plane-grating spectroscope or a prism spectroscope the natural place to introduce an occulting-bar is at the slit, for in both cases the slit is in focus on the plate both for horizontal and for vertical lines. There is of course some chance that a displacement of the slit will occur while the occulting-bar is being changed, but with moderate care such an accident is very unlikely.

In the case of a concave grating, owing to the great astigmatism, a horizontal line at the slit is very much out of focus on the plate, and therefore an occulting-bar at the slit would not occult. However, there is in front of the slit (i. e., on the side remote from the grating) a point for which a line at right-angles to the slit will be in focus on the plate. Consequently, it would seem that an occulting-bar placed at this point would secure the desired sharpness of focus, and at the same time be absolutely free from any possibility of causing accidental displacements in either plate or slit. The writer would be glad if this were seriously tried. At present he has not access to a concave grating, but some time ago he made a few preliminary trials with partial success. A bar about one-eighth inch wide was set up horizontally in front of the slit and a position found for which it was in approximate focus on the plate. The edges of the shadow did not appear very sharp; but whether this was due to lack of careful adjustment, to spherical aberration at the grating, or to diffused light from the grating (which was not a very good one) is not known. Of course a careful adjustment must be made to get the edges of the bar accurately straight and at right-angles to the slit, for the astigmatism of the grating affects the sharpness of this shadow, just as it affects the sharpness of the spectral lines. The bar may conveniently be made of the same shape as Rowland's shutter, but need be only two or three inches long—no longer, in fact, than the width of the beam of light at that point. Of course, the position of the bar would have to be changed whenever

the position of the grating is altered with reference to the slit; but this would cause less trouble than one might suppose, for in most cases of actual use the position of the grating is not frequently changed.

HERBERT M. REESE.

UNIVERSITY OF MISSOURI,
February 15, 1906.

A SHORT METHOD OF COMPUTING AN APPROXIMATE VALUE OF THE REDUCTION TO SUN IN RADIAL VELOCITY DETERMINATIONS

The chief function of the following method is to check roughly and quickly the rigorous computation, but it may also be used in reducing approximate measures of plates when an accuracy of 1 km is sufficient. In order to make it as simple as possible, the Earth is supposed to move in a circle with a constant velocity of 29.8 km per second. With this assumption the formula is reduced to

$$29.8 \cos\beta \sin(\odot - \lambda) \quad ,$$

where λ and β are the longitude and latitude of the star, and $\odot$ the longitude of the Sun. The factor $29.8 \cos\beta$ is constant for any star and may be determined once for all.

To avoid the same constants used in the rigorous computation, we may employ the time which has elapsed since conjunction with the Sun in longitude, instead of $(\odot - \lambda)$. If we let

$$T = \text{date of conjunction with Sun.}$$
$$t = \text{“ \quad “ observation.}$$

we have

$$(\odot - \lambda) = f(t - T) \ .$$

If T and t be expressed in days of the year, we shall have

$$(\odot - \lambda)^\circ = \frac{360}{365}(t - T)^{\mathrm{d}} .$$

If we now have a table giving $\sin\left\{\frac{360}{365}(t-T)\right\}$ for $(t-T)$ as argument, the whole operation becomes simplified to the following:

Subtract the date of conjunction (one of the constants of the star) from the date of observation, take out the sine with this as argument, and multiply this sine by $29.8 \cos\beta$ (the second constant).

A small error is introduced in using $(t-T)$, because of the eccentricity of the Earth's orbit. As most of the observations are made near opposition, it would be more accurate to use the date of opposition as

T, but this would introduce an opportunity for an error in the sign. As a compromise I have found a fictitious date of conjunction by first finding the date of opposition, and then subtracting $182^{d}.6$. Table II has been computed in this way. In computing, the dates were taken to tenths of a day, and then cut down to the nearest full day, as that is accurate enough and the constants will not change appreciably from year to year.

To test the accuracy of this method, the reduction to Sun was computed for several stars near the ecliptic at different times of the year and compared with the rigorous computations already made. The largest difference found was 0.6 km. As it was thought that this was not the largest difference possible, a test case was tried of a star on the ecliptic observed in October (when i has a maximum value) and only 30 days from conjunction. This gave a difference of 1.3 km, but for a star 150 days from conjunction, observed in October, the difference was reduced to 0.6 km, which was to be expected, as the tables are so computed as to make these differences a minimum in the vicinity of opposition. The average difference found for all actual observations was about 0.3 km.

For cases where T is greater than t—that is, when the preceding conjunction comes in the preceding year—we must add 365 to t, and the form becomes $(t+365-T)$. To avoid this extra addition a third column is given in Table II with the constant $365-T$, and when this is used it is to be *added* to the date of observation. For convenience we have been in the habit of writing the two on the card for the star, thus: 200−165, where the first is to be used if the observation and preceding conjunction occur in the same year, and the second when they occur in different years.

Table I facilitates finding the day of the year; Table II gives the dates of (fictitious) conjunction with the Sun for different longitudes; Table III gives the values of $29.8 \cos\beta$ for different latitudes; and Table IV gives the values of $\sin\left\{\frac{360}{365}(t-T)\right\}$. In this table the values for every tenth day are tabulated in the column headed "0," and those for the other days in the other columns.

As an example of the method of computing the reduction to Sun we will take the observation made on June 14, 1905, of λ *Andromedae:*

$$\begin{array}{rlrl} \lambda = & 16^\circ\ 54' & \beta = & +43^\circ\ 48' \\ t = & 165 & 29.8\cos\beta = & 21.4 \\ T = & 101 & & \\ t-T = & 64 \text{ days} & \sin f(t-T) = & 0.89 \end{array}$$

$$Va \qquad = +19.0 \text{ km}$$

$$Va \text{ rigorously computed} = +19.4$$

TABLE I

DAY OF YEAR

	Ordinary	Leap Year
January 0	0	0
February 0	31	31
March 0	59	60
April 0	90	91
May 0	120	121
June 0	151	152
July 0	181	182
August 0	212	213
September 0	243	244
October 0	273	274
November 0	304	305
December 0	334	335

TABLE II

λ	Date of Conjunction	λ	Date of Conjunction	λ	Date of Conjunction
0°	84^d—281	120°	203^d—162	240°	324^d— 41
10	94 —271	130	213 —152	250	334 — 31
20	104 —261	140	222^5—142^5	260	345 — 20
30	114 —251	150	232 —133	270	355 — 10
40	124 —241	160	242 —123	280	1 —364
50	134 —231	170	252 —113	290	11 —354
60	144 —221	180	262 —103	300	22 —343
70	154 —211	190	272^5— 92^5	310	32 —333
80	164 —201	200	283 — 82	320	43 —322
90	173 —192	210	293 — 72	330	53 —312
100	183 —182	220	303 — 62	340	63 —302
110	193 —172	230	313^5— 51^5	350	74 —291

TABLE III

β	$V \cos \beta$
0°	29.8 km
10	29.4
20	28.0
30	25.8
40	22.8
50	19.2
60	14.9
70	10.2
80	5.2
90	0.0

TABLE IV

$$\sin\left\{\frac{360}{365}(t-T)\right\}^{\circ}$$

$t-T$	0	1	2	3	4	5	6	7	8	9
0°	0.00	+0.02	+0.04	+0.05	+0.07	+0.09	+0.11	+0.12	+0.14	+0.16
10	+ .17	+ .19	+ .20	+ .22	+ .24	+ .26	+ .27	+ .29	+ .30	+ .32
20	+ .34	+ .36	+ .37	+ .39	+ .40	+ .24	+ .44	+ .46	+ .47	+ .49
30	+ .50	+ .52	+ .54	+ .55	+ .57	+ .58	+ .59	+ .60	+ .61	+ .63
40	+ .64	+ .65	+ .66	+ .67	+ .68	+ .69	+ .70	+ .72	+ .73	+ .74
50	+ .75	+ .76	+ .77	+ .78	+ .79	+ .80	+ .81	+ .82	+ .83	+ .84
60	+ .85	+ .86	+ .87	+ .88	+ .89	+ .90	+ .91	+ .92	+ .92	+ .93
70	+ .94	+ .95	+ .95	+ .69	+ .96	+ .97	+ .97	+ .98	+ .98	+ .99
80	+ .99	+ .99	+ .99	+1.00	+1.00	+1.00	+1.00	+1.00	+1.00	+1.00
90	+1.00	+1.00	+1.00	+1.00	+1.00	+1.00	+1.00	+1.00	+1.00	+ .99
100	+ .99	+ .99	+ .99	+ .99	+ .98	+ .98	+ .98	+ .97	+ .97	+ .96
110	+ .96	+ .95	+ .94	+ .93	+ .92	+ .91	+ .90	+ .90	+ .89	+ .88
120	+ .87	+ .86	+ .85	+ .85	+ .84	+ .83	+ .82	+ .81	+ .80	+ .79
130	+ .78	+ .77	+ .76	+ .75	+ .74	+ .73	+ .71	+ .70	+ .69	+ .68
140	+ .67	+ .65	+ .64	+ .63	+ .61	+ .60	+ .59	+ .58	+ .57	+ .55
150	+ .54	+ .52	+ .50	+ .49	+ .48	+ .47	+ .45	+ .43	+ .41	+ .40
160	+ .38	+ .37	+ .35	+ .33	+ .31	+ .30	+ .28	+ .26	+ .24	+ .23
170	+ .21	+ .19	+ .18	+ .16	+ .14	+ .12	+ .11	+ .09	+ .08	+ .06
180	+ .05	+ .03	+ .01	− .01	− .03	− .05	− .07	− .09	− .11	− .12
190	− .14	− .15	− .17	− .19	− .21	− .22	− .23	− .25	− .27	− .28
200	− .30	− .31	− .32	− .34	− .36	− .37	− .39	− .40	− .42	− .43
210	− .44	− .46	− .48	− .50	− .51	− .53	− .54	− .55	− .57	− .59
220	− .60	− .61	− .62	− .63	− .64	− .66	− .67	− .69	− .70	− .71
230	− .73	− .74	− .76	− .77	− .78	− .79	− .80	− .81	− .82	− .83
240	− .84	− .85	− .86	− .87	− .88	− .89	− .90	− .91	− .91	− .92
250	− .92	− .93	− .94	− .94	− .95	− .95	− .96	− .96	− .97	− .97
260	− .98	− .98	− .99	− .99	− .99	− .99	−1.00	−1.00	−1.00	−1.00
270	−1.00	−1.00	−1.00	− .00	−1.00	−1.00	−1.00	−1.00	−1.00	− .99
280	− .99	− .99	− .99	− .99	− .98	− .98	− .98	− .97	− .96	− .96
290	− .95	− .95	− .94	− .93	− .93	− .92	− .92	− .91	− .91	− .90
300	− .89	− .89	− .88	− .88	− .87	− .86	− .85	− .84	− .83	− .82
310	− .81	− .80	− .79	− .78	− .77	− .75	− .74	− .73	− .72	− .71
320	− .70	− .68	− .66	− .65	− .64	− .63	− .61	− .60	− .58	− .56
330	− .55	− .53	− .51	− .50	− .48	− .47	− .46	− .44	− .42	− .40
340	− .39	− .37	− .36	− .34	− .33	− .31	− .30	− .28	− .27	− .25
350	− .24	− .23	− .21	− .19	− .17	− .16	− .15	− .13	− .11	− .10
360	− .08	− .07	− .06	− .04	− .02	− .00				

H. K. PALMER.

MOUNT HAMILTON,
January 29, 1906.

LETTER FROM PROFESSOR J. LARMOR

BURLINGTON HOUSE, LONDON, W., June 28, 1906.

The Editor of the Astrophysical Journal.

DEAR SIR: Inclosed is a brief sketch of the result of the deliberations of the Royal Society regarding improved methods of publication of their journals. I think it will be of advantage to your readers to draw their attention to it as a piece of news, as we find that advertisements do not arrest attention. I need hardly say that what the Society desires in the first place is effective circulation, not least in America.

Very faithfully yours,

J. LARMOR,
Secretary, R. S.

Of the *Proceedings* of the Royal Society of London, as divided about a year ago into two series, Vols. 76–77 of Series "A," containing papers of a mathematical and physical character, and Vols. 76–77 of series "B," containing papers of a biological character, have now appeared, each running to about 600 pages royal octavo, with illustrations. A main object of this new arrangement was to render the *Proceedings* more accessible to workers by placing the two groups of subjects on sale separately, at a stated price attached to each separate part of a volume when it first appears. Moreover, with a view to promoting the circulation of the complete series it has been directed that a subscription paid in advance to the publishers, at the reduced price of 15*s*. per volume for either series, shall entitle subscribers to receive the parts as soon as published, or else the volumes when completed, in boards or in paper covers, as they may prefer.

With a view to further increasing the accessibility of the various publications of the Royal Society, each number of *Proceedings* now contains an announcement on the cover, of the more recent memoirs of the *Philosophical Transactions* as published separately in wrappers and the prices at which they can be obtained.

It is hoped that by this arrangement the difficulties which have been found to impede the prompt circulation of the journals of the Society, which are of necessity published in a somewhat different manner from a regular periodical, may be finally removed.

Reviews

Analysis of the Spectrum Lines of Mercury, Cadmium, Sodium, Zinc, Thallium, and Hydrogen; By L. Janicki.

This interesting research was presented as a dissertation at Halle in 1905, and appeared in the *Annalen der Physik*, **19,** 36–79, 1906.

The study of the composition of the lines in the spectrum of many elements has been the subject for many researches. The lack of good agreement between the different results led the author, as it has led others, to make further experiments.

An echelon grating made by A. Hilger was employed. It consisted of 32 plates 1 cm thick, with a step-width of 1 mm. The instrument seemed to be a very accurate one, giving very clear lines without ghosts, as is shown in the reproductions. The estimated limit of error in the measurement of the components was from 0.01 to 0.02 Å. U. for the very weak lines, 0.005 for the hazy lines, and 0.003 for the sharp lines.

The observations were made with many kinds of sources. The flame and the spark were not satisfactory, although by placing from six to ten large Leiden jars with sufficient self-induction in the secondary the sharpness of the radiations from the spark was much improved. He did not investigate the arc in air. For the mercury spectrum he employed the following sources: Arons' lamp of the form suggested by Lummer, large and small quartz lamps of Heraeus' make, Geissler tubes, vacuum tubes of the form used by Eder and Valenta, and tubes with external electrodes. The results as to the structure of the lines from all these sources were the same. In some cases the weaker components were invisible, due to the small intensity of the source. While the components often showed reversal and changes in their intensity, new satellites never appeared.

Mercury.—The results of Janicki's observations upon the mercury lines are as follows: The difference in wave-length of the components from the principal line is given in Å. U. Their relative intensity follows in brackets.

$\lambda = 5790.$

$+0.230$ ($\frac{1}{7}$); $+0.168$ ($\frac{1}{10}$); $+0.132$ ($\frac{1}{7}$); $+0.084$ ($\frac{1}{9}$);
Principal line (1); -0.119 ($\frac{1}{5}$); -0.187 ($\frac{1}{10}$); -0.251 ($\frac{1}{10}$).

$\lambda = 5769.$

+0.[illegible] ($\frac{1}{10}$); $+0.087$ ($\frac{1}{10}$); $+0.046$ ($\frac{1}{3}$);
Principal line (1); -0.050 ($\frac{1}{4}$); -0.113 ($\frac{1}{5}$).

$\lambda = 5461.$

$+0.133$ $(\frac{1}{8})$; $+0.088$ $(\frac{1}{3})$;
Principal line (1); -0.066 $(\frac{1}{7})$; -0.099 $(\frac{1}{10})$; -0.232 $(\frac{1}{5})$.
λ 4916 has no components.

$\lambda = 4359.$

$+0.121$ $(\frac{2}{3})$; $+0.105$ $(\frac{2}{3})$; $+0.043$ $(\frac{1}{4})$; $+0.020$ $(\frac{1}{5})$;
Principal line, (1); -0.023 (1); -0.052 $(\frac{1}{7})$; -0.097 $(\frac{2}{3})$;
-0.112 $(\frac{2}{3})$.

$\lambda = 4348.$

$+0.083$ $(\frac{1}{7})$; $+0.053$ $(\frac{1}{7})$; Principal line (1); -0.046 $(\frac{1}{6})$.

$\lambda = 4339.$

$+0.06$ $(\frac{1}{10})$; Principal line (1) -0.12 $(\frac{1}{10})$.

$\lambda = 4078.$

$+0.074$ $(\frac{1}{9})$; $+0.049$ $(\frac{1}{4})$; $+0.032$ $(\frac{1}{10})$;
Principal line (1); -0.046 $(\frac{1}{6})$; -0.076 $(\frac{1}{6})$.

$\lambda = 4047.$

$+0.067$ $(\frac{1}{2})$; Principal line (1); -0.051 $(\frac{3}{4})$; -0.111 $(\frac{1}{3})$

A very interesting and peculiar phenomenon occurred with the radiations from the quartz lamps. During the first few minutes after excitation the lines gave the constitution as given above. When their intensity was increased by taking out some of the external resistance, the yellow lines $\lambda\lambda$ 5790, 5769 and the bright green line λ 5461 broke up into five equidistant hazy lines. The same phenomenon happened in the radiations from the mercury spark in air. The most remarkable thing was that the number of these hazy lines or bands was always five. No explanation for this is offered.

Cadmium.—Both Michelson's tubes and tubes with external electrodes were used. The results are as follows:

$\lambda = 6439$ has no components.
$\lambda = 6325$ is a hazy line. It is a double line in tubes with external electrodes.
$\lambda = 5155$ is also a single line.

$\lambda = 5086.$

$+0.076$ $(\frac{1}{5})$; Principal line (1); -0.026 $(\frac{1}{8}?)$

$\lambda = 4800.$

$+0.059$ $(\frac{1}{4})$; Principal line (1); -0.034 $(\frac{1}{3})$; -0.080 $(\frac{1}{5})$.

$\lambda = 4678.$

$+0.030$ $(\frac{1}{3})$; Principal line (1); -0.056 $(\frac{1}{6})$.
$\lambda = 4662$ is a single line.

Sodium.—Due to the high temperature in the cadmium tubes, sodium vapor was given off from the glass. The D lines were without components, but they easily reversed, giving the appearance of double lines.

Zinc.—A tube with external electrodes was employed. The lines $\lambda\lambda$ 6362, 5182, 4810, 4722, 4680, are all sharp and single.

Thallium.—The green line of thallium λ 5351 produced in a Hamy tube has one component of half the intensity of the principal line and of greater wave-length.

Hydrogen.—The red hydrogen line λ 6563 is double, the distance between the components is 0.14 Å. U. Both components show reversal.

The following are his conclusions. They cannot be considered as new. His results in general corroborate those of other investigators, and his remarks regarding the causes of the differences in the results have been cited before.

1. All the strong lines in the mercury spectrum are very complex, with the exception of λ 4916. Only some of the lines of cadmium have components. The D lines and the zinc lines are single. The red line of hydrogen is double.

2. No change in the wave-length of the components was observed. The relative intensities, however, often varied. This result is worthy of notice, because such changes give the appearance of a shifting of the lines if the apparatus has not sufficient resolving power.

3. "Ghosts" may be excluded from the observations. Even if the agreement of the above results with those of others is not good, the author has good reasons for the exclusion of ghosts. In all parts of the spectrum the echelon used showed different strong lines as single, while Lummer and Gehrcke with their spectroscope did not find a line without components. Further proof is given by the very different structure of the neighboring lines $\lambda\lambda$ 5790, 5769 of mercury, 5896, 5890 of sodium, and 5876 of helium. To this is added the good agreement of the measurement of the yellow helium line with that of Runge and Paschen, who used a large Rowland grating.

4. The larger the resolving power of a spectroscope, the easier can a reversal of a line be observed which as such is often impossible to recognize.

JAMES BARNES.

JOHNS HOPKINS UNIVERSITY.

Atlas Stellarum Variabilium, Series V, containing, for all parts of the sky, the variable stars whose minimum light is above magnitude 7. By J. G. HAGEN, S.J., Director of Georgetown College Observatory. Berlin: Felix L. Dames, 1906. 10×12 inches, 21 charts, catalogue and index.

Series I, II, and III of this important work, already published, contain the well-known telescopic variables from declination $-25°$ to the north pole. Series IV, promised for the autumn of 1906, will comprise a list of the brighter telescopic variables for which a three-inch telescope will suffice. The approaching completion of this useful work thus adds to the value of each part. It is also of interest to note that this is probably the last work of the kind which will be done without the aid of photography.

From the nature of the subject-matter, the charts and catalogue sheets differ in numerous particulars from the former series. As most of the charts contain more than one variable, they are arranged by regions, and therefore on different scales, the radius of the sphere varying (Table I) between 160 and 650 mm. The stars are printed in black with disks of different shapes and carefully graded sizes from magnitudes 1 to 7. The lines and constellation names on the charts are in red, therefore indistinct by lamplight, in fact almost invisible by the red light which many observers prefer for recording observations of faint variables at the telescope.

The difficult problem of deciding just what stars to show on the charts has been solved as follows. Only stars to the fifth magnitude are shown, except in the vicinity of the variables. In the vicinity of each variable, fainter stars to magnitude 7 were selected *from the sky itself*, for the double purpose of furnishing a suitable series of comparison stars and making identifications easy and certain. As a further aid the sheet for Chart XIX has in one corner a small chart of the region around η *Carinae* on a threefold scale. While more of these auxiliary charts would be useful, it is evident that completeness would be an impossibility.

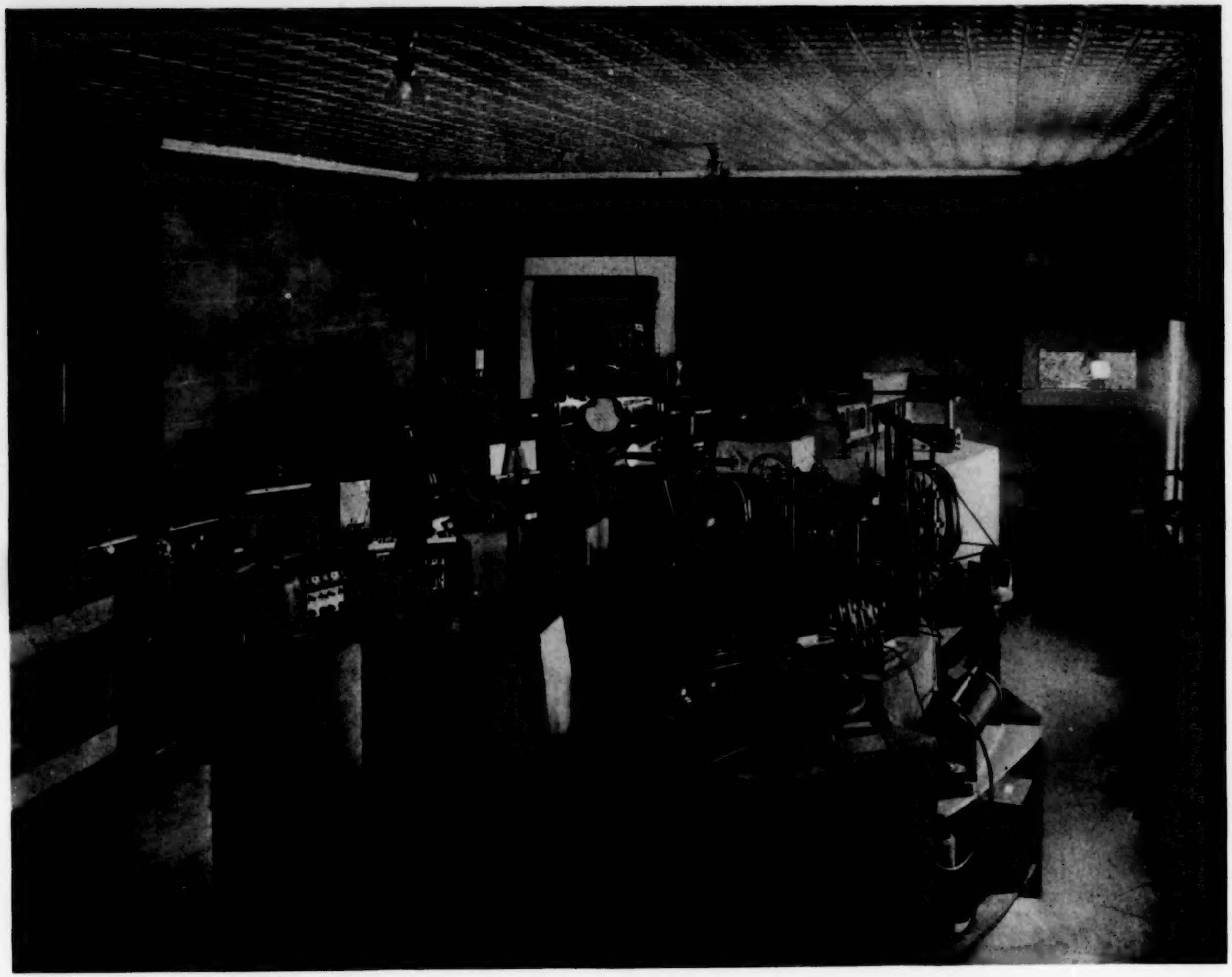
The catalogue sheets accompanying each chart are models. They contain the current number of each star shown on the chart, the constellation, Bayer's letter, the Flamsteed and *B. D.* numbers, and the positions for 1900. Then follow the most important columns, giving the photometric magnitudes from the Potsdam and Harvard catalogues, or for the southern stars from the *Uranometria Argentina*, a column of notes giving other designations of some stars and colors from Krueger's *Catalog der farbigen Sterne*. Last, and most important of all, is a table for each

variable showing the comparison stars which have been used by each of the best observers who have worked with this class of variable stars.

Only one variable of doubtful standing has been admitted to the list, that being *32* (*RU*) *Cassiopeiae*, for which the evidence for and against a change in light is conflicting and very puzzling. But this does not detract from the high praise which must be accorded to this exceedingly valuable work which puts the astronomical world under great obligations to the distinguished author and his beneficent patroness, the late Catherine W. Bruce, by whose liberality the expenses of publication were met

J. A. PARKHURST.

PLATE I



SPECTROSCOPIC LABORATORY OF THE SOLAR OBSERVATORY